Physics 1140 Fall 2013
Introduction to Experimental Physics

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Lecture 3:

Error propagation

Correlated/uncorrelated errors

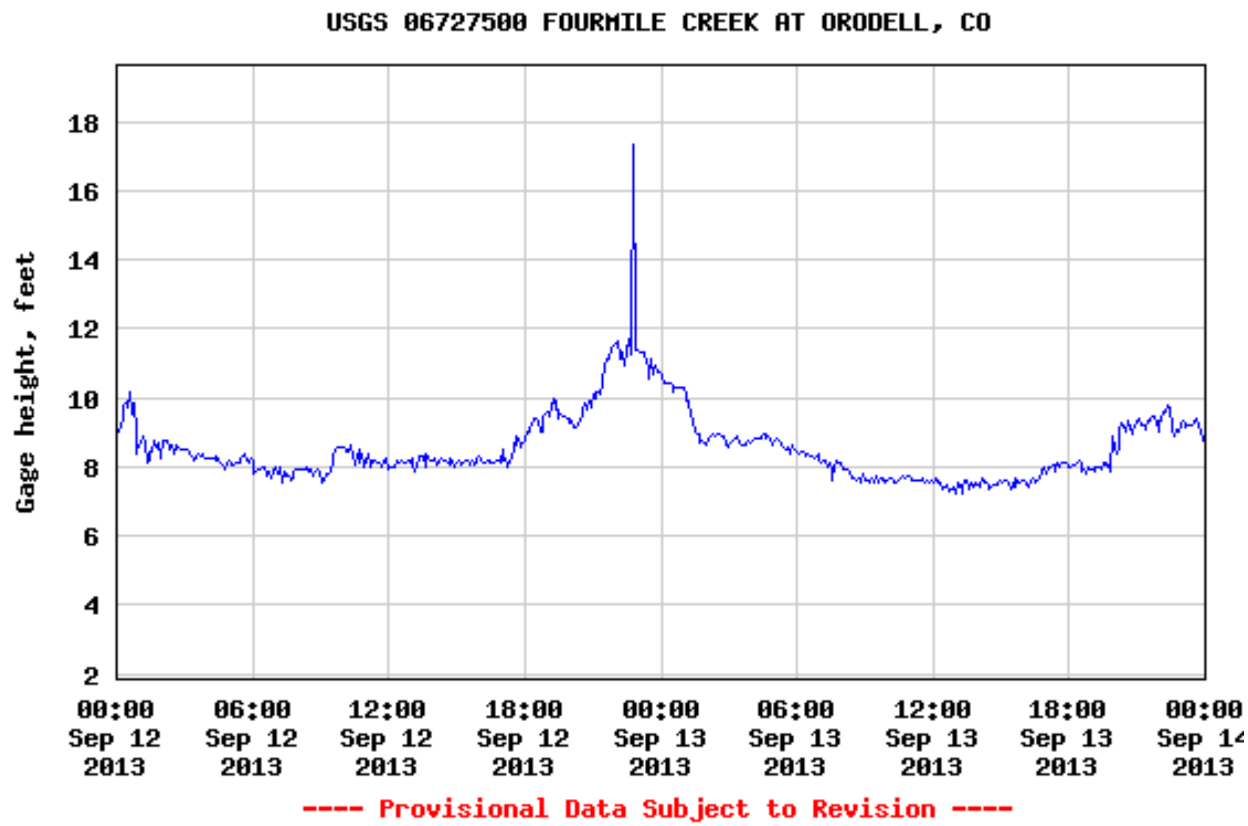
Experiments

- How do we determine what is a good measurement/ calculation?
- What is reasonable?
- Do we have something to compare our measurement to?
- Is our comparison significant?

- Errors detected by making repeated measurements
- Reduced by testing and calibrating equipment
- Average or mean reduces data variations, finds a central value
  - more on this next week

Boulder Creek data

	Discharge cubic ft/s	Gauge height (ft)
22:00	1,180	11.58
22:05	1,200	11.65
22:10	1,070	11.12
22:15	1,080	11.13
22:20	1,130	11.34
22:25	1,040	10.95
22:30	1,130	11.37
22:35	1,210	11.69
22:40	1,100	11.23
22:45		17.35
22:50	1,180	11.58
22:55	1,130	11.36
23:00	1,140	11.41
23:05	1,120	11.30
23:10	1,120	11.30
23:15	1,120	11.30
23:20	1,040	10.98
23:25	1,030	10.91
23:30	947	10.57
23:35	1,070	11.12



http://waterdata.usgs.gov/nwis/uv?cb_00060=on&cb_00065=on&format=gif_default&period=1&begin_date=2013-09-06&end_date=2013-09-13&site_no=06727500

Discrepancy

Discrepancy = the difference in the magnitude of two measured quantities.

$$Discrepancy = |A - B|$$

where A and B are two measured values or a measured and theoretical value, etc.

Suppose A = (3.2 $\pm$ 0.2)m/sec and B = (5.3 $\pm$ 0.2)m/sec.

What is the discrepancy?

Is that discrepancy significant?

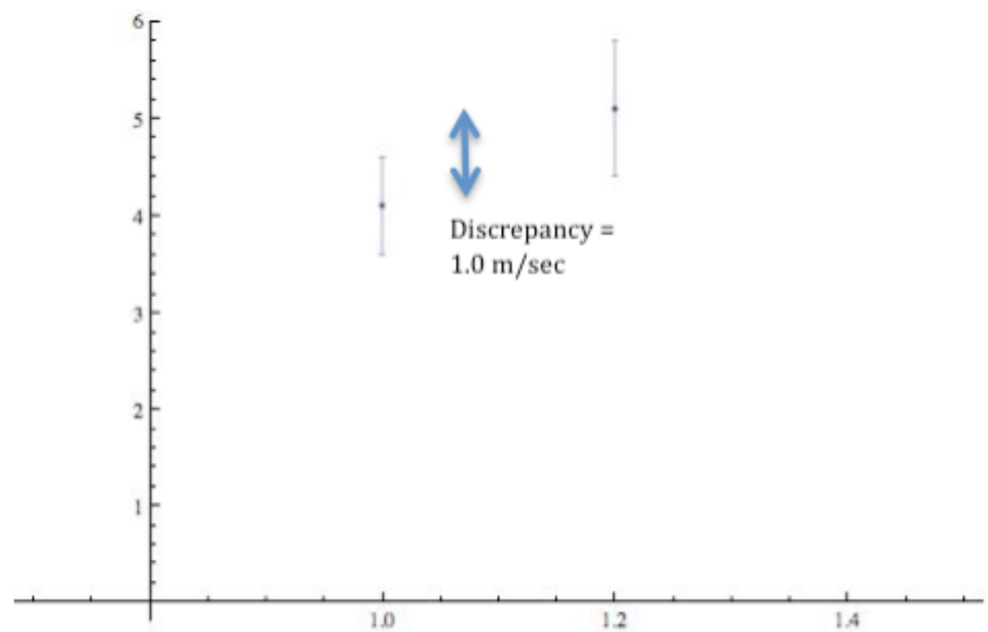
Discrepancy is significant

(3.2 ± 0.2)m/sec

(5.3 ± 0.2)m/sec

(4.1 ± 0.5)m/sec

(5.1 ± 0.7)m/sec



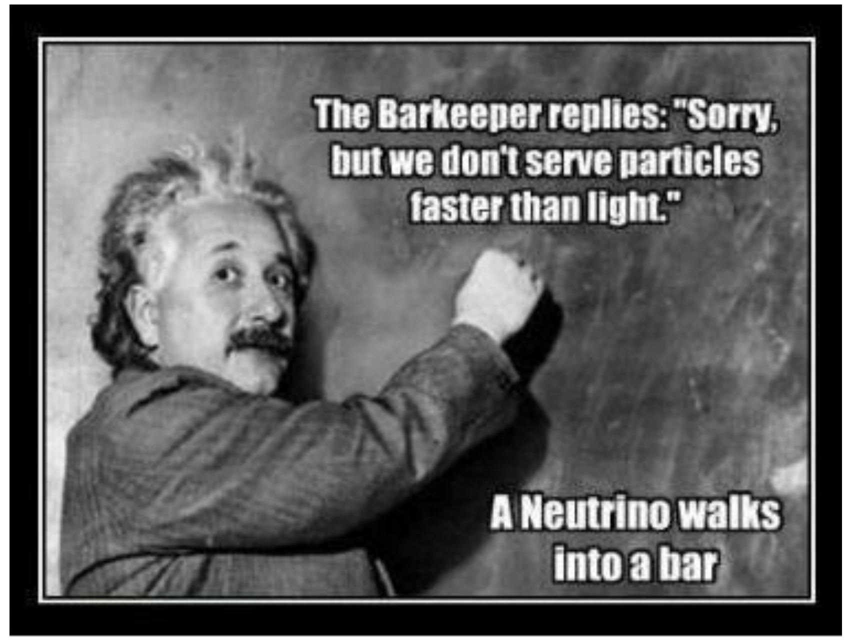
Clicker question 1

Two research groups make a measurement of a new elementary particle. The two measurements are $m_1 = (7.8 \pm 0.1) \times 10^{-27}$ kg and $m_2 = (7.0 \pm 0.2) \times 10^{-27}$ kg. Based on the reported masses, did the two research groups measure the mass of the same particle? Explain.

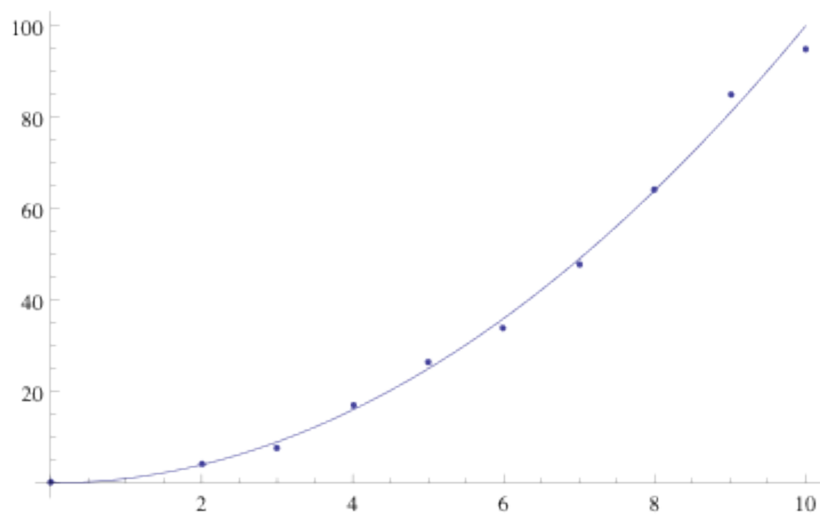
- A. Yes, the discrepancy is only 0.8×10^{-27} kg which is very small so they must have measured the same particle
- B. No, the discrepancy is large, such that the uncertainties of either measurement do not overlap so the discrepancy is significant.
- C. No, the uncertainties are much too large which means the measurements are too imprecise to be considered at all.
- D. Yes, measuring small quantities is very hard to do and any measurements that are even remotely close to each other indicates each group measured the same particle.

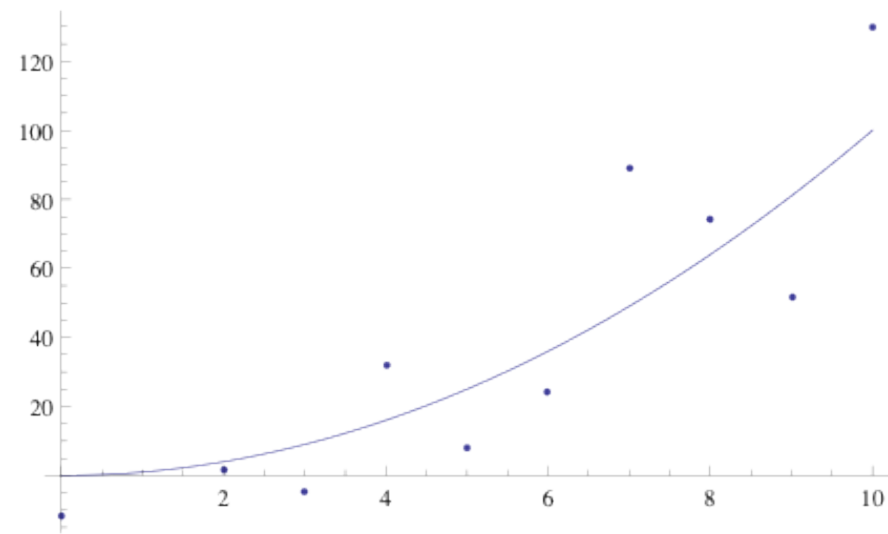
What if the discrepancy is significant

- Design flaw in procedure or experiment set up.
- Under/overestimate the uncertainty in measurements.
- Calibration of instruments.
- Mistakes made while performing experiment?
  - do it over!
- Redesign and start over.



Random vs. Systematic Errors

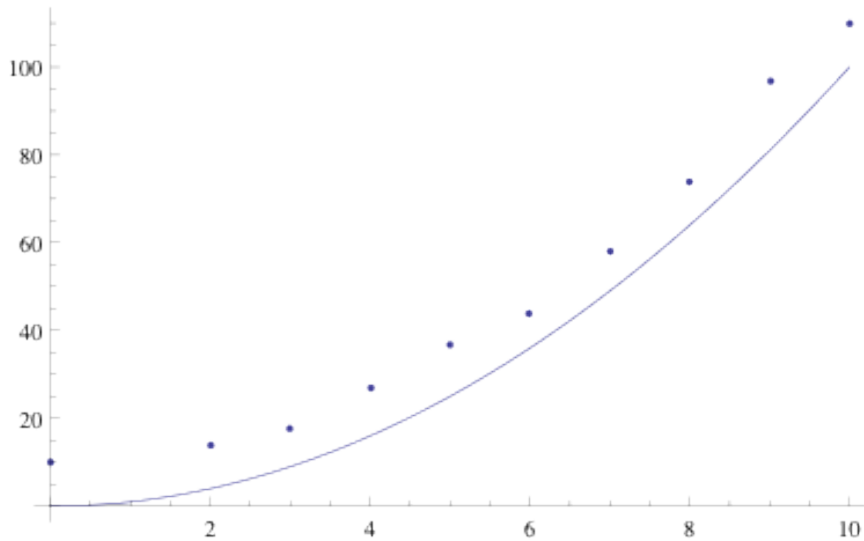




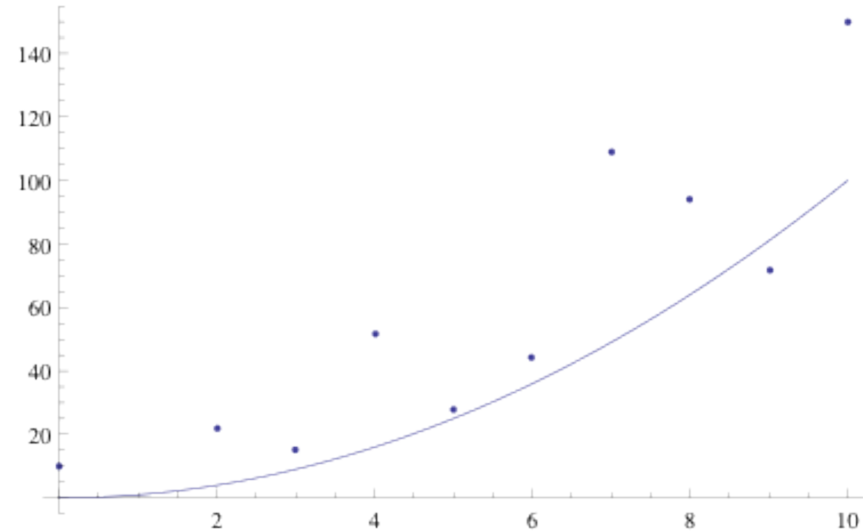
RANDOM: LOW
SYSTEMATIC: LOW

RANDOM: HIGH
SYSTEMATIC: LOW

Random vs. Systematic Errors



RANDOM: LOW
SYSTEMATIC: HIGH



RANDOM: HIGH
SYSTEMATIC: HIGH

Error propagation: Master rule

- Suppose you measure variables x, y, z … and calculate some function of these measurements $f(x, y, z \ldots)$
- What is the uncertainty δf? As long as the errors in x, y, z … are independent/uncorrelated, then

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\delta z)^2 + \ldots}$$

Notation

- $\frac{df}{dx}$ = full derivative with respect to x (i.e. f(x))
- $\frac{\partial f}{\partial x}$ = partial derivative with respect to x (i.e. f(x,y,z...))
- δf = ("delta f") uncertainty in f

Clicker question 2

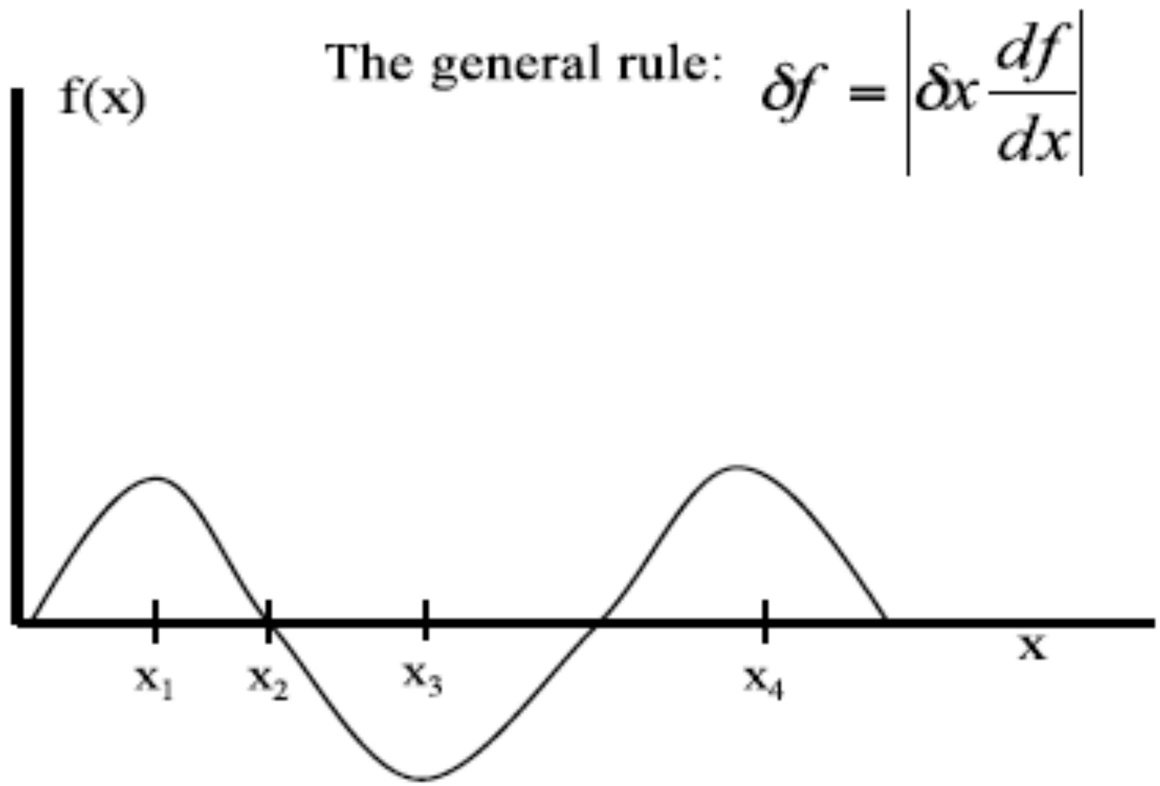
For what value of x is δf the largest? (δx is the same in all four cases.)

A: x_1

B: x_2

C: x_3

D: x_4



Uncertainties

Suppose that x, . . . , z and u, . . . , w are measured values with uncertainties, and are used to compute:

$$q = \frac{x \times \ldots \times z}{u \times \ldots \times w}$$

If the uncertainties are independent and random, then the fractional uncertainty is the sum in quadrature of the original fractional uncertainties:

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \ldots \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \ldots + \left(\frac{\delta w}{w}\right)^2}$$

So it's sometimes useful to think of uncertainty in terms of the fractional error $\delta x / x$

Examples

- $f = xy/z$

$$\frac{\partial f}{\partial x} = \frac{y}{z},\ \frac{\partial f}{\partial y} = \frac{x}{z},\ \frac{\partial f}{\partial z} = \frac{-xy}{z^2}$$

$$\frac{\delta f}{f} = \sqrt{\left(\frac{y/z\delta x}{f}\right)^2 + \left(\frac{x/z\delta f}{f}\right)^2 + \left(\frac{-xy/z^2\delta z}{f}\right)^2}$$

$$= \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2}$$

Clicker question 3

What is the algebraic expression for the absolute error δE of the kinetic energy, $E = ½mv^2$, given uncertainties in the mass (δm) and velocity (δv)? of

A: $$\delta E = \sqrt{\left(\frac{1}{2}v^2\delta v\right)^2 + (mv\delta m)^2}$$

B: $$\delta E = \sqrt{\left(\frac{1}{2}v^2\delta m\right)^2 + (mv\delta v)^2}$$

C: $$\delta E = \sqrt{\left(\frac{1}{2}v^2\delta v\right)^2 + (mv\delta v)^2}$$

D: $$\delta E = \sqrt{\left(\frac{\partial E}{\partial m}\delta v\right)^2 + (\frac{\partial E}{\partial v}\delta m)^2}$$

Clicker question 4

$x = 20.0 \pm 0.2$ cm, $y = 5 \pm 1$ cm, $z = 8.0 \pm 0.4$ cm

$V = xyz$

Roughly how much is the fractional error in volume, $\delta V/V$?

A. A part in a thousand

B. About 1%

C. About 5%

D. About 20%

E. Something different.

Uncertainties

Uncertainties in a power, derived from the "Master Rule"

$q = x^n$ where n is a fixed, known number

If the uncertainties are independent and random, then the fractional uncertainty is:

$$\frac{\delta q}{|q|} = |n| \frac{\delta x}{x}$$

Uncertainties in trig functions: make sure your uncertainties are in radians

Correlated errors

A mysterious, generous banker takes a big pile of pennies, cuts it exactly in two, and gives one pile to Frank, one to Jill. He tells them the piles are exactly, to the penny, the same, but he doesn't tell them how many there are.

- Frank does a rough count of his pile, gets $F = 200 \pm 30$.
- Now he knows that Jill's share $J = 200 \pm 30$!
- Note that δJ and δF are perfectly correlated. If "rough
- count" of F is too high, then "rough count" of J is too high, also!

Clicker question 5

- In the above scenario, J = F, F = 200 ± 30, difference D = J – F, we can say that D ± δD is

A) 0 ± 0 pennies

B) 0 ± 30 pennies

C) 0 ± 42 pennies

D) 0 ± 60 pennies

E) some other answer

Clicker question 6

- In the same scenario, J = F, F = 200 ± 30, sum S = J + F, we can say that $S \pm \delta S$ is

  A) 400 ± 0 pennies

  B) 400 ± 30 pennies

  C) 400 ± 42 pennies

  D) 400 ± 60 pennies

  E) some other answer

Negatively (or anti-)correlated errors

Now our banker counts out exactly 1000 pennies, not one more or one less.

- He offers some to Frank, who takes a big scoop of them.
- He gives the rest to Jill, telling her there were originally exactly 1000 pennies.
- Jill does a "rough count" of her pennies, and estimates J = 200 ± 30
- With the "exactly 1000" info Frank must have F = 800 ± 30.
- In this case δJ and δF are perfectly anti-correlated. If "rough count" of J is too high, then "rough count" of F is too low!

Clicker question 7

- In the above scenario, $J + F = 1000$, $J = 200 \pm 30$, difference $D = F - J$, we can say that $D \pm \delta D$ is

A) 600 ± 0 pennies

B) 600 ± 30 pennies

C) 600 ± 42 pennies

D) 600 ± 60 pennies ←

E) some other answer

Uncorrelated errors

This time the banker takes a huge sack of pennies, thousands and thousands, and offers them to Frank.

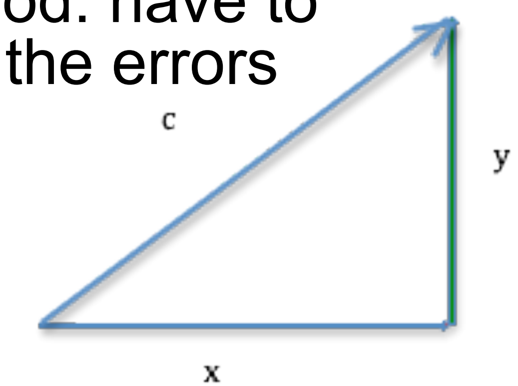
- Frank politely takes only a handful. He does a rough count and gets $F = 300 \pm 30$
- Then Jill gets a turn. She dips into the huge sack with both hands, pulls up a quantity of pennies. She does a rough count of her pile and gets $J = 700 \pm 30$
- This time, δF and δJ are nothing to do with each other.
- They are "statistically independent", or "uncorrelated."

Error propagation: correlated errors

- What if the errors in variables x, y, z, … are not independent, but correlated?
- What is the uncertainty σ_f? Standard method: have to estimate cross-terms involving products of the errors (covariance).

$$mag(c) = \sqrt{c^2} = \sqrt{(x+y)*(x+y)}$$

$$= \sqrt{x^2 + y^2 + 2x*y}$$



- Easier: add up uncertainties ("worst case scenario"). Gives a reasonable upper bound on the total uncertainty σ_f.

$$\sigma_f = \left|\frac{\partial f}{\partial x}\right|\sigma_x + \left|\frac{\partial f}{\partial y}\right|\sigma_y + \left|\frac{\partial f}{\partial z}\right|\sigma_z + \ldots$$

- But virtually always (at least for our purposes here), we can assume variables are independent.

Next week

- Minimizing the effects of random errors through repeated measurements
- Gaussian distribution
- Mean and standard deviation

- Starting second Mechanics lab next week – prelabs due immediately before start the lab
- Monday should now be on same schedule as rest of week
- First homework due Wednesday 4pm (G2B66)
- New homework due next Wednesday.