Physics 1140 Fall 2013

Introduction to Experimental Physics

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Lecture 2:

Uncertainties

Clicker question 1

- Which option is NOT in standard format?

A: 6260 ± 20 s

B: 3.174 ± 0.036 m/s

C: 0.382 ± 0.003 m

D: $(3.01 \pm 0.03) \times 10^{-3}$ gal

value ± uncertainty [units]
1 significant figure in uncertainty
same precision for value and uncertainty

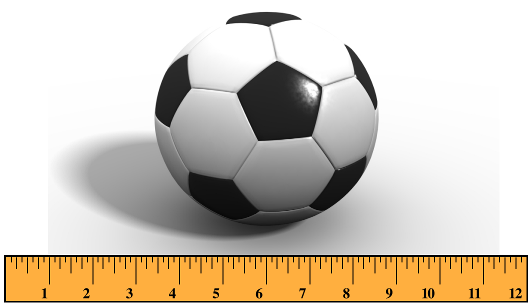
Today

- More on uncertainties
  - systematic and random/statistical
- Precision vs accuracy
- Discrepancy: comparison with theory/other measurement
- Propagating error
  - errors on derived quantities
- Homework 1 will be posted on Wednesday, due next Wednesday, 4pm

More on uncertainties

- Previously we talked about uncertainties estimated based on resolution
  - how precise the measuring instrument is.

- But there can be other sources of error!
  - environmental, calibration, drift, parallax, observer error...

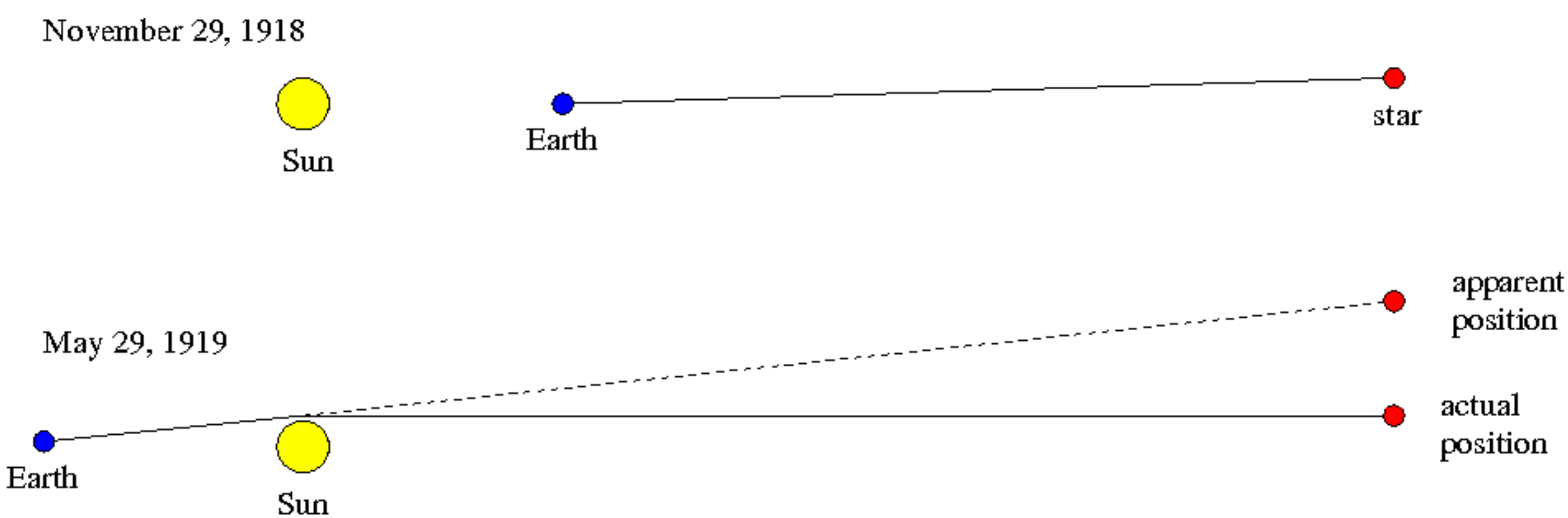


- Systematic error: aspects of the experiment which bias the results
  - Always the same amount, in the same direction.

- Random errors: random fluctuations which vary from measurement to measurement. (Often called "statistical" error.)

Example: light bending

- Theory proposed by Einstein in 1911
  – prediction based on General Relativity: 1.75 arcseconds
  – prediction based on "Newtonian mechanics": 0.875 arcseconds (Soldner, 1803)
- First test in 1919, during a solar eclipse



http://planetarium.lambuth.edu/

Example: light bending

- During solar eclipse, moon obscures Sun, revealing field of stars around it
  - Take pictures with Sun and without and compare

- Experimental details:
  - Two expeditions, to West Africa and South America, so two sets of data to compare – but different telescopes and different magnifications

    ← SYSTEMATIC ERRORS!
  - Turbulence in Earth's atmosphere causes star light deflection ← RANDOM ERRORS!

Systematic errors

- Miscalibration of equipment, e.g. instrument with faulty markings, different magnifications!
  - Try to re-calibrate equipment (if possible.)
- Parallax error in reading an analog display – same angle every time.
- Observer bias (e.g. “jumping the gun” in a timing measurement.)

- Systematic effects can be subtle and hard to identify. The usual approach is to minimize (or eliminate) them when possible.

Random errors

- A random error defines the typical spread in a measurement caused by random effects.
  - This effect could have a real physical cause:
    - e.g. Thermal fluctuations bending light.
  - It can also be the result of the measurement technique:
    - Timing experiment: random fluctuations in reaction time
    - Parallax error: if don't read scale from the same vantage point each time.
- Solutions:
  - Use a larger number of samples!
  - Often errors are on the order of an instrument's precision (e.g. measuring mm with a meter stick)
    - Use a more precise tool, if possible.

Accuracy vs precision

- Systematic and random errors are related to the concepts of accuracy and precision in measurement.
- Accuracy: how close are your experimental results to some hypothetical “true” value?

$$\text{accuracy (\%)} = 100 \cdot \frac{|\text{exp.} - \text{true}|}{\text{true}}$$

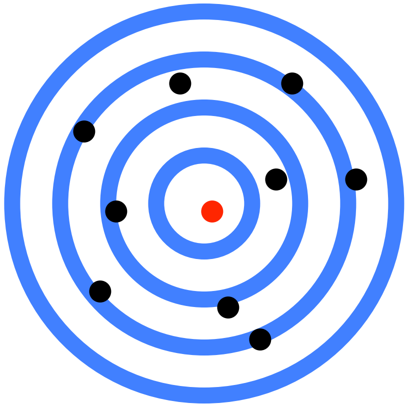
- Precision: how large is the spread in your data or the uncertainty of the measurement? Smaller spread = more precise measurement.
- Discrepancy: How well does your data agree with expectation?

$$\text{discrepancy} = |\text{exp} - \text{true}|$$

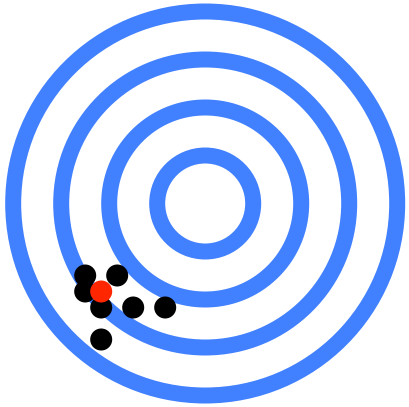
$$\text{or } |\text{exp} - \text{true}| / \text{uncertainty}$$

Accuracy vs precision

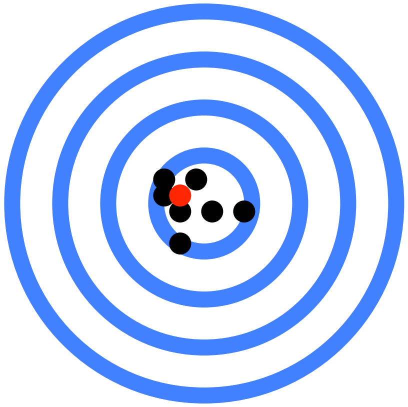
- Goal: measurements should be both accurate and precise.



Accurate: mean value in bullseye. Imprecise: spread in values is large.



Inaccurate: mean value far from bullseye. Precise: spread in values is small.

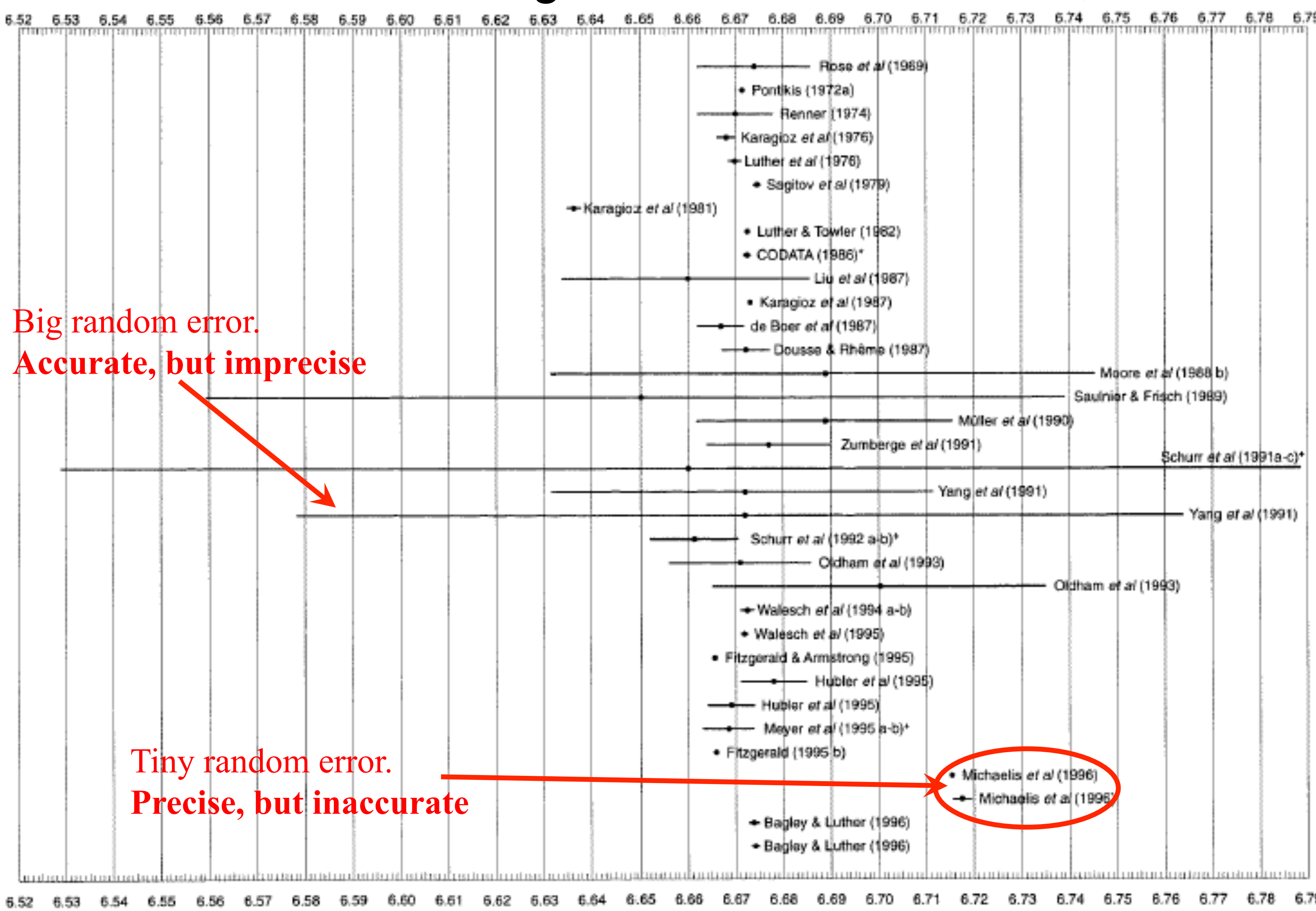


Accurate: mean value in bullseye. Precise: spread in values is small.

- Accuracy is affected by systematic shifts. Precision can be improved by reducing random errors – i.e. increasing the number of measurements.

Real experiment: measuring *G*

Observations of Newton's gravitational constant *G*, 1969 – 1998.



* See Cohen and Taylor (1987).

+ The error bars represent the quadrated sum of the individually listed Type A and Type B uncertainties.

G.T. Gillies, Meas. Sci. Technol. **10** (1999) 421-425

Example: light bending

- 1.6 ± 0.3 arcseconds

$$\text{Accuracy} = 100 \cdot \frac{|1.60 - 1.75|}{1.75} = 8.5\%$$

$$\text{Precision} = 100 \cdot \frac{0.3}{1.60} = 19\%$$

$$\text{Discrepancy: } |\text{expt} - \text{true}| = |1.60 - 1.75| = 0.15 \approx 0.5\sigma$$

- 1.98 ± 0.12 arcseconds

$$\text{Accuracy} = 100 \cdot \frac{|1.98 - 1.75|}{1.75} = 13\%$$

$$\text{Precision} = 100 \cdot \frac{0.12}{1.98} = 6\%$$

$$\text{Discrepancy} = |1.98 - 1.75| = 0.23 \approx 2\sigma$$

Clicker question 2

Four measurements of G yield the values

A: $G = (6.681 \pm 0.005) \mathrm{x} 10^{11}\ \mathrm{Nm^2kg^{-2}}$

B: $G = (6.67 \pm 0.02) \mathrm{x} 10^{11}\ \mathrm{Nm^2kg^{-2}}$

C: $G = (6.658 \pm 0.008) \mathrm{x} 10^{11}\ \mathrm{Nm^2kg^{-2}}$

D: $G = (6.7 \pm 0.1) \mathrm{x} 10^{11}\ \mathrm{Nm^2kg^{-2}}$

Which measurement is the most precise?

Clicker question 3

The accepted value is 6.674×10^{11} Nm^2kg^{-2}.

A: $G = (6.681 \pm 0.005) \times 10^{11}$ Nm^2kg^{-2}

B: $G = (6.67 \pm 0.02) \times 10^{11}$ Nm^2kg^{-2}

C: $G = (6.658 \pm 0.008) \times 10^{11}$ Nm^2kg^{-2}

D: $G = (6.7 \pm 0.1) \times 10^{11}$ Nm^2kg^{-2}

Which measurement is most accurate?

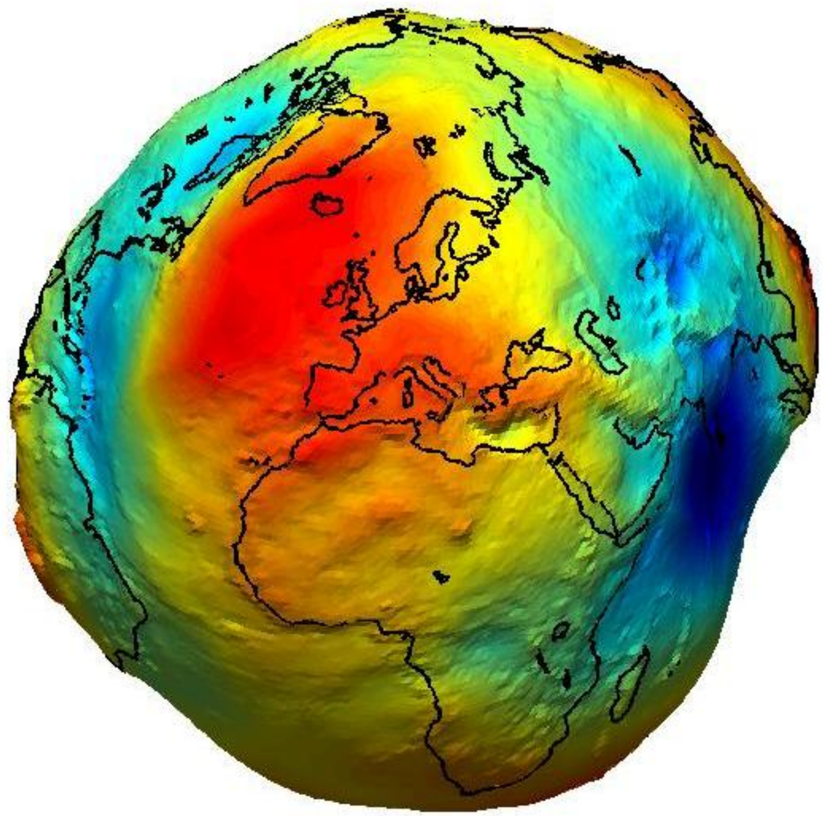
Why precision is important

- If measurements are sufficiently precise, we can observe changes in g due to tides, mantle convection, earthquakes, density of nearby material, etc.

$$g = 9.780318(1 + 0.005302\sin^2 L - 0.000006\sin^2 2L) - 3.086\cdot 10^{-6} h$$

$h =$ height above sea level (m)

$L =$ latitude



Gravity and Steady-State Ocean Circulation Explorer (GOCE)

Figure: geoid, constant g surface around Earth (image radially distorted).

Uncertainties

- Uncertainty in a Measured Quantity Times and Exact Number, i.e.

$$q = Bx$$

where B is an exact number and x is a measured quantity with some uncertainty δx:

$$\delta q = |B| \delta x$$

If q = 2x and x = 8.21 ± 0.04
then q = 16.42 ± 0.08

Uncertainties

- Uncertainties on adding and subtracting measured quantities
  - $x \pm \delta x$, $y \pm \delta y$, and $q = x + y$
- Could just add the uncertainties, but this will probably provide an overestimate of the uncertainty
- If the uncertainties are independent and random, the total uncertainty is the quadratic sum

$$q = x + \ldots + z - (u + \ldots + w)$$

$$\delta q = \sqrt{(\delta x)^2 + \ldots + (\delta z)^2 + (\delta u)^2 + \ldots + (\delta w)^2}$$

Clicker Question 4

4. If f = 2.39 $\pm$ 0.02 m and x = 8.21 $\pm$ 0.04 m, calculate g = f + x along with the error and put your answer in Standard Format.

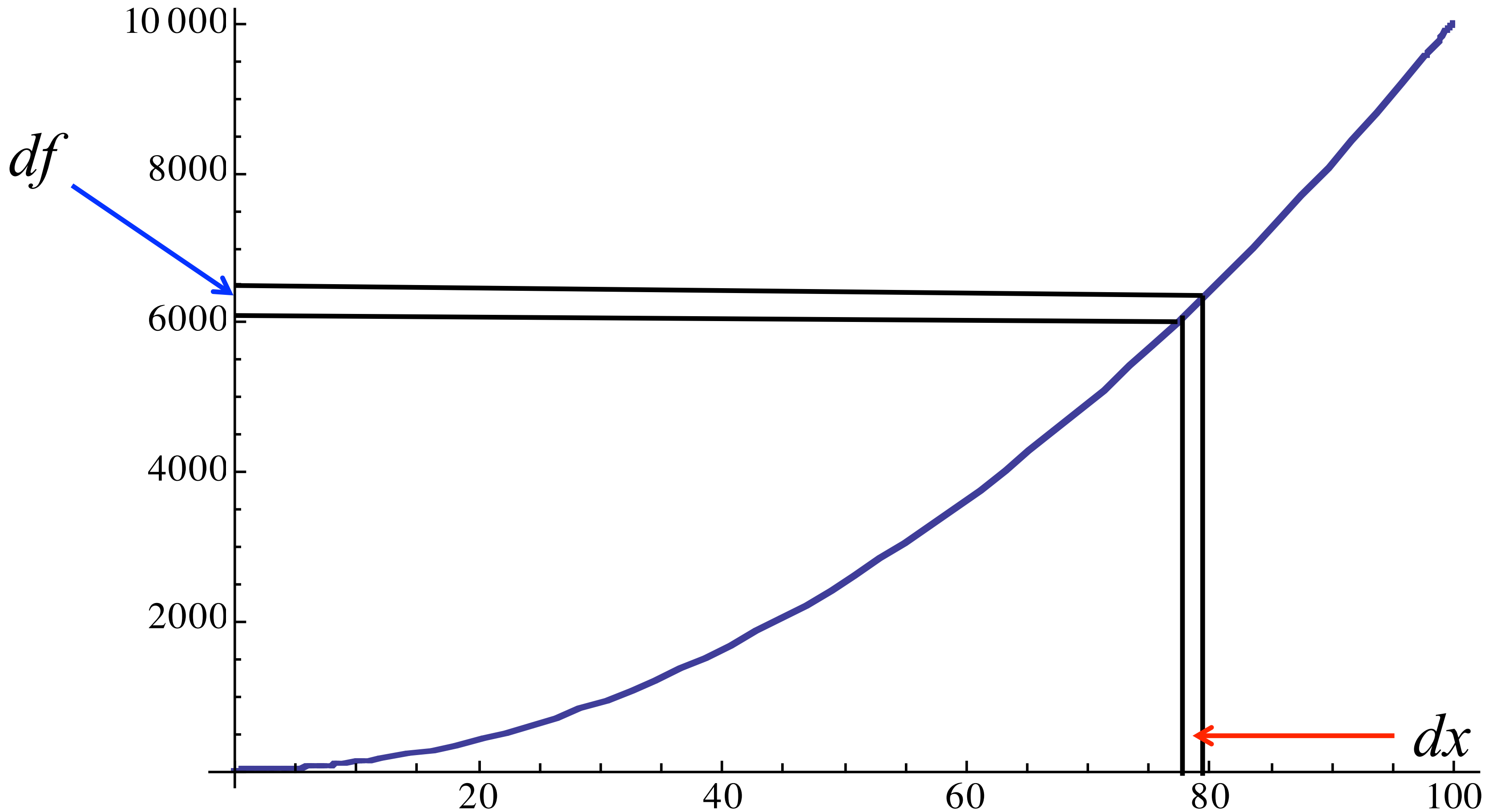
- A. 10.6 $\pm$ 0.04 m
- B. 10.60 $\pm$ 0.044 m
- C. 10.60 $\pm$ 0.04 m
- D. 10.60 $\pm$ 0.06 m

Propagation of errors

- Suppose you measure a number x with some uncertainty δx.
- You then want to calculate a quantity which is a function of x: $f(x)$.
  - eg. you measure the radius of a circle and want to find the area.
- There must be some uncertainty in f, δf due to the uncertainty δx.
- How do we calculate δf? This is the subject of error propagation.

Uncertainties on functions

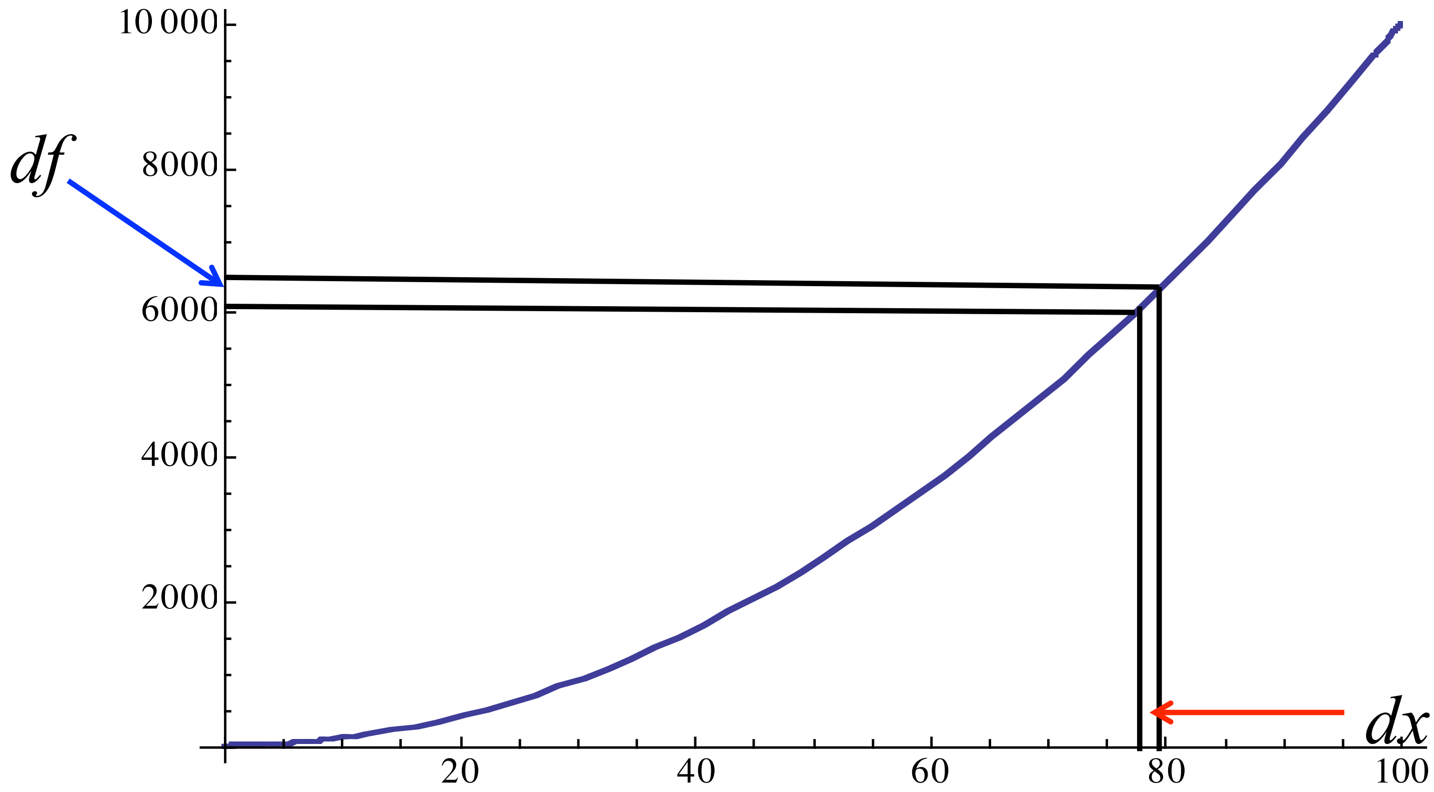
$f(x) = x^2$



Uncertainties on functions

$f(x) = x^2$ and e.g. $x = 10.0 \pm 0.5$

Then $\delta f = |f(x) - f(x+\delta x)| \approx |10^2 - 10.5^2| \approx 10$



Error propagation: one variable

- One variable: If the uncertainty δx is relatively small, then δf is given by

$$\delta f = \left|\frac{df}{dx}\right| \delta x$$

- Example: you measure the radius of a circle; what is its area?

$$r = 10.0 \pm 0.3 \text{ cm}$$

$$A = \pi r^2 = 100 \cdot \pi \text{ cm}^2 \approx 314 \text{ cm}^2$$

$$\delta A = \left|\frac{dA}{dr}\right| \delta r = (2\pi r)\delta r = (2\pi \cdot 10 \text{ cm})(0.3 \text{ cm}) \approx 19 \text{ cm}^2$$

$$A = 310 \pm 20 \text{ cm}^2$$

Error propagation: several variables

- Suppose you measure variables x, y, z … and calculate some function of these measurements $f(x, y, z \ldots)$

- What is the uncertainty δf? As long as the errors in x, y, z … are independent/uncorrelated, then

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\delta z)^2 + \ldots}$$