Problem Set 5 – Phys 7810 – Spring 2011

Due date: Th 3 March

Understanding the linear and nonlinear optics of the 2-level system with damping might well be the most important aspect of this class. Despite its simplicity its behavior and limiting cases are unusually rich. It is the starting point for the description of a wide range of spectroscopic phenomena.

1. *Optical response of 2-level system*: In this problem you shall fill in the details in the derivation we sketched in class for the derivation of the expectation value for the induced polarization for a 2-level atom. Hence, start with $i\hbar\frac{\partial\rho_{01}}{\partial t} = [\hat{H}, \hat{\rho}]$, derive the equations of motion for the density matrix elements including both dephasing T_2 and population relaxation times T_1. Then, for the steady-state response, apply the rotation-wave approximation for near-resonance excitation, and derive $P(t) = \epsilon_0\chi E$ for this case.

2. *Expansion of optical response of 2-level system*: In this problem we want to analyze the optical response of the 2-level system in leading order in a perturbative expansion. Start with the analytic solution of the driven 2-level system in the rotating wave approximation, and its macroscopic polarization for an ensemble with number density N/V. From that derive the perturbative expression for the leading order susceptibilities. Discuss which terms describe what kind of processes, such as absorption, gain, gain saturation, self-phase modulation, etc. Notice and discuss the absence of even order terms, and the absence of third-harmonic generation.

3. *Strong and weak field response of 2-level system*: Here we want to numerically study the time evolution of the induced dipole moment for a two-level system and compare to the analytic expression in the rotating wave approximation.

   In particular we are interested in the limiting cases of a highly-nonlinear behavior, for the underdamped case, i.e., where the inverse Rabi frequency is much larger than T_1 and T_2. Consider a system with resonance frequency ω_{10} and dipole operator μ_{01}. Use a step excitation for the driving field.

   Now solve the differential equations and plot $\langle\tilde{\mu}(t)\rangle$ and its Fourier spectrum $\langle\tilde{\mu}(\omega)\rangle$. For comparison plot the analytic solution in the rotating wave approximation for the following two cases: i) weak field case on resonance ($\Delta = 0$) and moderate driving force ($\frac{1}{T_1\omega_{01}}, \frac{1}{T_2\omega_{01}} \ll \frac{\Omega}{\omega_{01}} \ll 1$), and ii) slightly off-resonance ($\frac{1}{T_1\omega_{01}}, \frac{1}{T_2\omega_{01}} \ll \frac{\Delta}{\omega_{01}} \ll 1$) with Rabi frequency at appreciable level, e.g., $\Omega \simeq 0.01\omega_{01} = \sqrt{(\frac{\Omega_0}{\omega_{01}})^2 + (\frac{\Delta}{\omega_{01}})^2} \ll 1$. Take each $\ll$-symbol to correspond to a factor of 10.

   For each case, increase the drive frequency and/or drive detuning and examine the behavior and deviations from the analytic solution. When does the rotating wave approximation fail? Discuss its failure and the new phenomena that arise in this highly non-perturbative regime.