Problem Set 4 – Phys 7650 – Nonlinear Optics – Spring 2015

Due date: Fr 13 March

1. *Self-phase modulation*: In our discussion in class on self-phase modulation we have assumed a symmetric pulse and instantaneous material response leading to the symmetric temporal evolution for the spectral shift.

   a) Discuss and sketch the spectral result you expect if you were to use a asymmetric pulse, such as a (rounded) sawtooth, both in time- and frequency domain.

   b) Now assume a relaxation, i.e., damped behavior for the response of the index of refraction change with respect to the incident field of the form

   $$\tau_r \frac{dn_{NL}}{dt} + n_{NL} = n_2 I(t) \tag{1}$$

   with relaxation time τ_r. Use a symmetric pulse of the form $I(t) = I_0 \text{sech}^2(t/\tau_0)$. Calculate (numerically, or analytically) the nonlinear phase shift and change in instantaneous frequency. Discuss briefly the limiting cases for the relaxation time τ being shorter or faster than the pulse duration τ_0.

2. *Nonlinear Schrödinger equation*: In this problem you shall go through the derivation of the Nonlinear Schrödinger equation starting with the nonlinear wave equation:

   $$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = 0. \tag{2}$$

   (Note: the nonlinear response is buried in the nonlinear refractive index n). We only sketched the general strategy in class, and I gave some intermediate results. Use your notes to guide you through the process where you shall arrive at:

   $$\frac{\partial E_0(z,t')}{\partial z} + \frac{1}{2} i k_2 \frac{\partial^2 E_0(z,t')}{\partial t'^2} = i\gamma \left|E_0(z,t')\right|^2 E_0(z,t'). \tag{3}$$

   Resort to the literature as little as possible where of course you can find the derivation in full detail. How is the final answer modified if you were not to make the approximation $k^2 - k_0^2 \simeq 2k_0(k - k_0)$?

3. *Optical soliton*: Show that a pulse of the form $E_0(z,t') = A_0 \text{sech}(t/\tau_0) e^{i\kappa z}$ is a solution of the pulse propagation eq. (3) above. How do pulse duration τ_0 and amplitude A_0 have to be related to each other? Show that the phase shift of the pulse is given by $\kappa = -k_2/2\tau_0^2$. Discuss the general requirements for k_2 and γ for soliton formation.

4. *Absorption coefficient from first principles*: Use the quantum description we derived for the first order susceptibility to estimate the absorption coefficient on resonance for an atomic vapor assuming a number density of 10^{17} cm^{-3}, dipole transition moment $\mu = 1.5 e a_0$, resonance wavelength of 500 nm, and spectral linewidth (FWHM) $\Delta\nu/\nu$ of 10 GHz.

5. *Refractive index from first principles*: Similarly, estimate the refractive index of glass in the visible, i.e., off-resonance. Choose realistic values for the atomic number density, dipole transition moment, and detuning from resonance.