Problem Set 3 – Phys 7650 – Nonlinear Optics – Spring 2015

Due date: Fr 24 February

1. Derive the expression for the tuning curve for an OPO for, e.g., its signal wave $\omega_s(\theta)$ for the case of e-pump, and o-signal and o-idler.

2. *Optical parametric generation*: Using an input beam of wavelength 400 nm to pump an optical parametric process in a BBO crystal (use data from last hw sheet), we can obtain a wavelength tunable coherent output beam by angle-tuning of the crystal to achieve phase matching. Consider the problem qualitatively at first: what tuning range do you expect for signal and idler beam? Assume extraordinary pump, ordinary signal, and ordinary idler, what type of phase matching does that correspond to? Then calculate and plot the tuning curve $\lambda(\theta)$.

3. *Phase sensitivity of parametric amplification*: The phase Φ of the laser fields participating in the parametric generation and amplification process can play an important role as you will see in this problem: Consider a strong pump field $E_p = E_{0,p}e^{i(k_p z - 2\omega t)}$ and signal field $E_s = E_{0,s}e^{i(k_s z - \omega t + \Phi)}$ incident on a nonlinear optical crystal under phase matched conditions $k_p = 2k_s$ for the degenerate case. Solve the coupled amplitude equations:

$$\frac{\partial E_s}{\partial z} = 2i\frac{\omega_s}{cn_s}\chi^{(2)}E_pE_s, \text{ and } \frac{\partial E_p}{\partial z} = 2i\frac{\omega_s}{cn_p}\chi^{(2)}E_s^2, \tag{1}$$

   considering Φ. Show that for $\Phi = 0$ or π the signal is amplified exponentially, whereas for $\Phi = \pi/2$ the signal is attenuated. This result shows the nature of phase sensitive amplification. Only the in phase components get amplified, and the out of phase (or quadrature) components are damped. This effect allows for the generation of non-classical light (squeezed states).

4. *Optical parametric oscillator*: In class we showed that the fields E_s and E_i in a phase matched parametric amplifier with an undepleted pump E_p grow as:

$$E_s(z) = E_s(0)\cosh(\gamma z) + i\sqrt{\frac{\omega_s n_i}{\omega_i n_s}}E_i(0)\sinh(\gamma z), \tag{2}$$

$$E_i(z) = E_i(0)\cosh(\gamma z) + i\sqrt{\frac{\omega_i n_s}{\omega_s n_i}}E_s(0)\sinh(\gamma z). \tag{3}$$

   Consider an OPO with crystal of length L, with reflectivities R_s and R_i for both cavity mirrors, and power absorption loss per unit length of α. In steady state the gain is balanced by the loss per round trip.

   a) Derive an expression for the threshold gain g_0 as a function of the reflectivities and absorption coefficient.

   b) Show that for the doubly resonant OPO ($1 - R_1 \ll 1, 1 - R_2 \ll 1$) with no absorption the threshold is given by

$$g_0^2 = \frac{(1-R_1)(1-R_2)}{L^2}. \tag{4}$$

   c) Show that for a singly resonant OPO ($1 - R_1 \ll 1, R_2 \ll 1$) with no absorption the threshold is given by

$$g_0^2 = \frac{R_2(1-R_1)}{L^2}. \tag{5}$$

   d) For $R_1 = 0.99$ find the threshold intensity needed for the OPO in (c). Assume $\omega_s \sim \omega_i \sim 1\mu$m, $d_{\text{eff}} \sim 10$ pm/V, and $n_s \sim n_i \sim n_p \sim 2$.