Problem Set 2 – Phys 7650 – Nonlinear Optics – Spring 2015

Due date: Fr 13 February

1. *First second-harmonic generation:* The story goes that the editorial office has removed the specs in the photograph the authors send in of their claimed first second-harmonic observation - and with them the evidence of the first ever observation of SHG: Franken *et al.* Phys. Rev. Lett. **7**, 118 (1961). Calculate with the parameters give in the paper and the values below the intensity you expect the authors would have observed and compare with their estimate as given in the paper. Use: quartz $d_{11} = 0.37$ pm/V, $n_0(694\text{ nm}) = 1.54084$, $n_0(347\text{ nm}) = 1.5660$, $n_e(694\text{ nm}) = 1.5496$, $n_e(347\text{ nm}) = 1.5760$. Assume all other nonlinear coefficients to be 0 due to Kleinman symmetry.

   First draw the two possible configurations he describes in his paper and calculate the coherence length l_c for the interaction. Use this to estimate what focus size (1/e^2 intensity) he has focused his laser beam to achieve the "interaction volume" he specifies.

   Calculate the conversion ratio for the peak intensity of his beam and use this to estimate the maximum amount of SH power you would expect. How does your result compare with Franken's estimate of SH photons produced?

2. A simple problem in the infrared: Find the phase-matching angle for second-harmonic generation whenCO2 laser radiation of wavelength 10.6?m isincident on a uniaxial tellurium crystal. Given: $n_o(\omega)$=4.796, $n_e(\omega)$=6.243, $n_o(2\omega?)$=4.856.

3. *Sum-frequency generation*: Sum-frequency generation (SFG) can be used to up-convert infrared photons from a spectral range where detectors might be insensitive to higher energy visible SF-photons. In the infinite plane-wave approximation, starting from the equation of phase-matched SFG

$$\frac{\partial E_s}{\partial z} = i\frac{2\omega_s}{cn_s}\chi^{(2)}E_{vis}E_{IR} \tag{1}$$

   and approximating $E_{vis}(z) \simeq E_{vis}(0)$, and knowing that the number of SF photons generated are equal to the number of IR photons converted, derive the expression for the SF intensity $I_s(L) = f(\omega_s, \omega_{IR}, \omega_{vis}, I_{IR}, I_{vis}, L, n_s, n_{IR}, n_{vis}, \chi^{(2)})$. Hint: on the way you should arrive at an expression of the form $dy/dz = a(1-y^2)^{1/2}$ with solution $y = sin(az)$. From the result show that for a proper choice of L and I_{vis} you can obtain a maximum photon conversion efficiency from IR to vis of 1.

4. Walk-off: Due to the double refraction, the e- and o-beams in a birefrigent crystal experience a walk-off from each other. This limits the useful crystal length in frequency conversation, e.g., SHG.

   a) For a uniaxial crystal, calculate the walk-off angle between e- and o-beam.

   b) Show that for type-I phase matching (o+o → e).

$$\rho \approx \tan\rho = \frac{\mathrm{n}_0^2(\omega)}{2}\left\{\frac{1}{\mathrm{n}_\mathrm{e}^2(2\omega)} - \frac{1}{\mathrm{n}_\mathrm{o}^2(2\omega)}\right\}\sin(2\theta) \tag{2}$$