Lab M2. Ultrasound: Interference, Wavelength, and Velocity

The purpose of this exercise is to become familiar with the properties of waves: frequency, wavelength, phase, and velocity. We use ultrasonic waves because the wavelength is easily measured with an ordinary meter stick. The frequency of ultrasound is above the range of human hearing, so the experiment does not create an audible sound. The experiment also illustrates the interference of waves

Sound is a pressure wave in air. When we hear a sound, we are sensing a small variation in the pressure of the air near our ear. The speed of a sound wave in air is about 340m/s or about 5 seconds to travel one mile, and this speed depends only on the properties of the air (temperature, composition, etc.) and not on the frequency or wavelength of the wave.

Consider a sinusoidal sound wave in air with frequency f and wavelength λ. The speed v is related to f and λ by

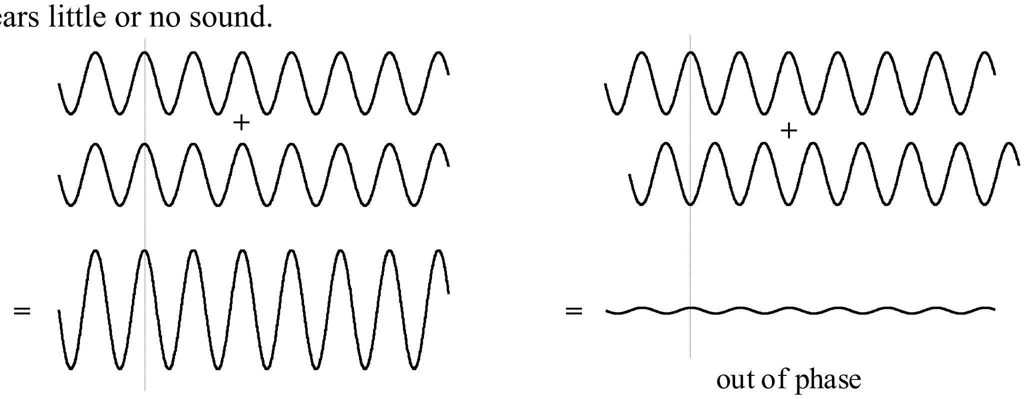
(1) $$v = f\,\lambda.$$

To see where this relation comes from, think:

$$\text{speed} = \frac{\text{change in distance}}{\text{change in time}}$$

The time it takes for one wavelength of the sound to go by is the period T, so $v = \lambda/T$. But $f = 1/T$ so $v = \lambda f$. Note that as f increases, λ goes down, but the speed v stays the same. The frequency range of human hearing is about 20 Hz to 20,000 Hz. (The upper end drops as we age; for people over 60, it is about 12 kHz, while dogs can hear up to about 35 kHz.)

Consider two sound waves of equal f, equal λ, and nearly equal amplitude, both approaching a detector, such as a human ear. If the two waves arrive at the ear in phase, that is with successive maxima arriving at the same time and successive minima arriving at the same time, then the waves interfere constructively, their amplitudes add, and the ear hears a loud sound. But if the waves arrive at the ear exactly out of phase, that is, with the maxima of one wave arriving at the same time as the minima of the other wave, then the waves interfere destructively; they cancel and the ear hears little or no sound.

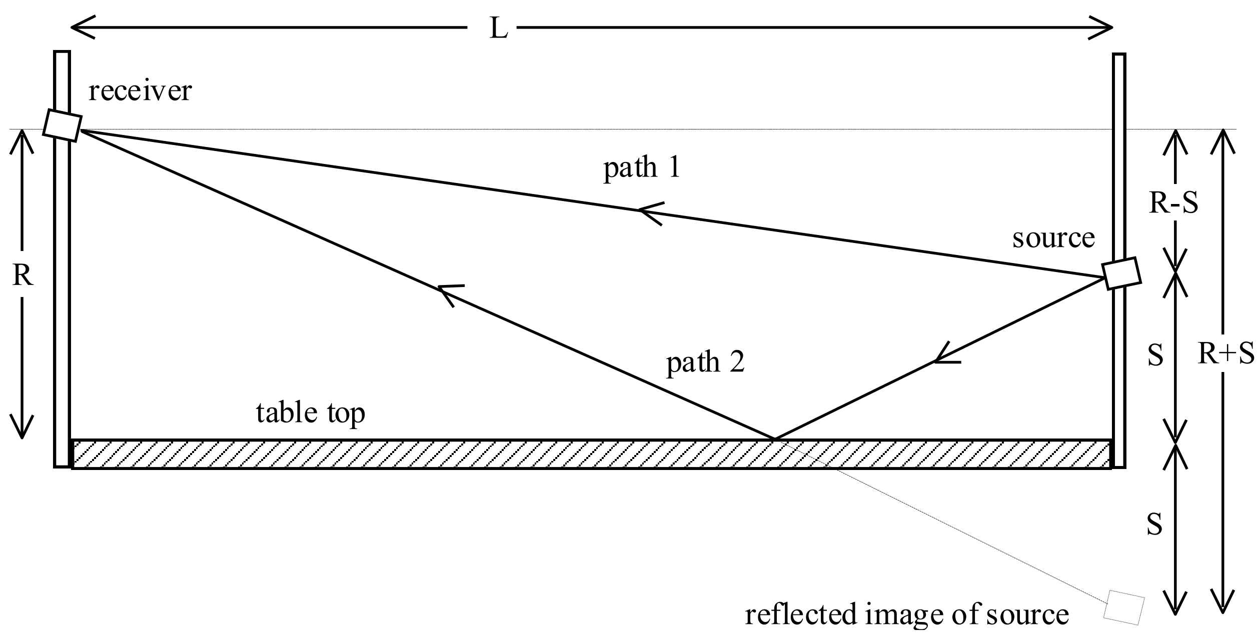


in phase

out of phase

Now, in order to observe the interference of sound waves in the way just described, the two waves must have exactly the same frequency. One way to insure that the two waves have the same frequency is to arrange to have both waves generated by the same source. This is the principle of the so-called Lloyd's Mirror arrangement, shown in the diagram below. A source of sound — a speaker emitting a pure tone with known frequency f — sits a height S above a flat table. A receiver sits a distance R above the table, a distance L along the table away from the source. Sound from the source can travel to the receiver along two different paths: the sound can travel directly from the source to the receiver (path 1 with total length D_1) or the sound can reflect from the surface of the table to the receiver (path 2 with total length D_2). (Sound, like light, can reflect from a smooth flat surface with the angle of incidence equal to the angle of reflection.) The receiver "sees" a reflection of the speaker in the table top which appears to be at a distance D_2 .

Whether the two waves arrive at the receiver in phase or out of phase depends on the path difference ($D_2 - D_1$). If the path difference is an integral number of wavelengths ($D_2 - D_1 = n\,\lambda$) then the waves arrive in phase. If $D_2 - D_1 = (n + \frac{1}{2})\,\lambda$, then the waves arrive out of phase and the detector receives a small amplitude total wave. In this lab, you will measure the heights R and S at which interference maxima and minima occur. From this information, you will compute the wavelength λ of the sound. Finally, from the wavelength and the known frequency f, you will compute the speed of sound $v = \lambda\, f$.



From the diagram above, we see that the two path lengths are

(2) $$D_1 = \sqrt{L^2 + (R-S)^2} \text{ and } D_2 = \sqrt{L^2 + (R+S)^2}\ .$$

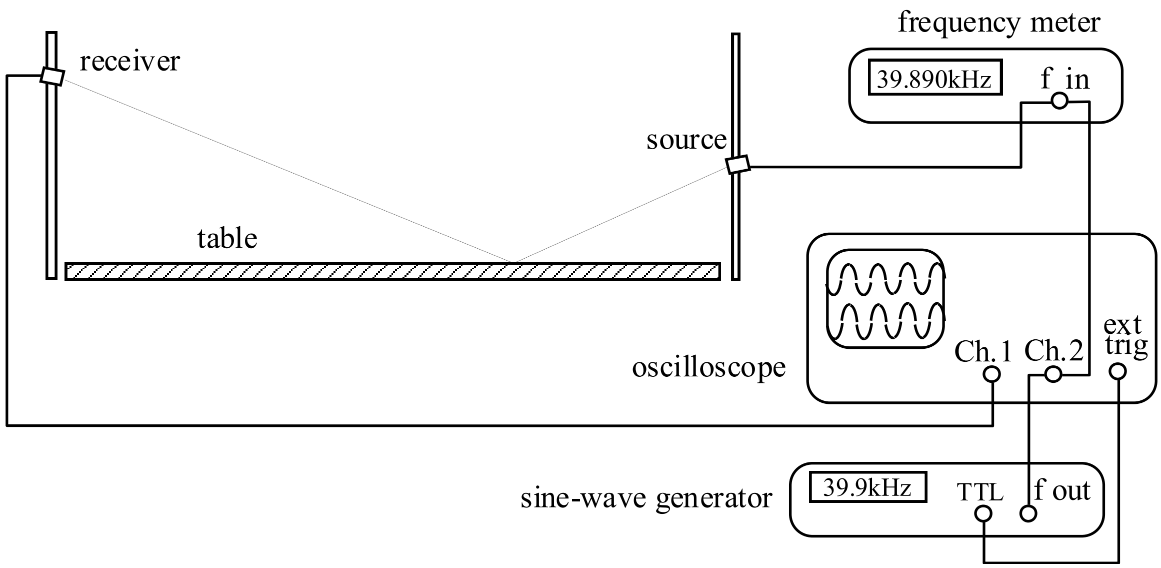
Interference minima occur at the receiver when

(3) $$D_2 - D_1 = \sqrt{L^2 + (R+S)^2} - \sqrt{L^2 + (R-S)^2} = (n + \tfrac{1}{2})\,\lambda, \qquad n = 0,1,2,3,\ldots$$

Hence, if n is known, then one can compute λ from measurements of R, S, and L.

Experiment

A schematic of the apparatus is shown below. The sound source and receiver are ultrasonic transducers which are tuned to operate at about 40 kHz, well above the range of human hearing. The receiver signal is displayed on an oscilloscope, allowing the user to see when the signal is maximum or minimum.



Begin by turning everything on and allowing the sine-wave generator and the frequency meter to warm up for several minutes and stabilize. Aim the source directly at the receiver and tune the frequency for a maximum signal around 40 kHz. The source and receiver are tuned transducers and do not work well if only slightly away from the optimum frequency. Record this frequency f and, from time to time during the experiment, check whether f has drifted at all. Retune to the original frequency if necessary.

Part 1. Wavelength Measurement

Set the source at some height around 20 cm above the table, aiming it approximately at the opposite edge of the table. Measure the distance S from the table top to the center of the source and L, the distance between the source and receiver. Now move the receiver up and down along its support. You should see the output on the oscilloscope go through several maxima and minima as you move the receiver.

Place the receiver level with the top surface of the table and slowly raise it until you encounter the 1st minimum. This should be the n=0 minimum, corresponding to a path difference of $\frac{1}{2}\lambda$. Record the height R = h_o of the n=0 minimum. Continue slowly raising the receiver, recording the heights h_1, h_2, h_3, .. of the subsequent minima. Record as many minima as possible and then make a plot of h_n vs. n.

Looking at eq'n (3) for inspiration, define a Mathematica list for λ, which is the wavelength computed from the height h at which the n^{th} minimum occurs. Make a plot of λ vs. n using the ListPlot[] function. Recall that, in Mathematica, you can use the Thread[] function to combine your lists for λ and n so that they can be plotted easily.

If there are no systematic errors, the computed wavelength λ should be independent of n. If you see a λ gradually increasing or decreasing with n, it is a sure sign of a systematic error, most likely an inaccurate measurement of the zero position (table position) of the receiver. It is sometimes found that the λ's computed for the first few smallest n's differ substantially from the others. This is because, when h is small, the two paths D_1 and D_2 are almost the same and there is a large fractional uncertainty in the difference (D_2-D_1). From your plot of λ vs. n, decide which data points, if any, should be eliminated from further analysis. Using your "good" data, compute the mean, standard deviation, and standard deviation of the mean of λ.

$$\lambda avg = (1 / N) * \left(\sum_{i=1}^{N} \lambda[[i]]\right)$$ (Average wavelength)

$$\sigma = \sqrt{\left(\left(\sum_{i=1}^{N} (\lambda[[i]] - \lambda avg)^2\right) \Big/ N - 1\right)},$$ (Standard Deviation)

$$\sigma mean = \sigma \Big/ \sqrt{N}$$ (Standard Deviation of the Mean)

Part 2. Speed of Sound

Using the known frequency f, and your measured value for λ, compute the speed of sound v. Also compute the uncertainty δv, from the uncertainties in λ and f.

Near room temperature, the speed of sound in air depends on the temperature according to

(4) $$v(m/s) = 331.5 + 0.607 \cdot T$$

where the temperature T is in degrees Celsius (there is a big dial thermometer on the wall in the lab). Compare this known v with your measured v . Find the difference between your measured v from Eq(1) and the known v from Eq(4) and compare this difference to your calculated uncertainty δv.

Discuss the results, commenting on the agreement/disagreement of the theory with the experiment, the most significant sources of uncertainty in the measurement and how these might be improved.

PreLab Questions:

1. If the speed of sound is v = 345 m/s, what is the range of wavelengths of sound which the human ear can detect?

2. (Counts as two questions.) Show how you will define the wavelength λ in your Mathematica document. [Just write the Mathematica definition like it will appear on the computer screen, except write it on paper with your pen.] Also show how you will make a Mathematica graph of λ vs. n. For instance, if I want to show how to define a function $y(x,t) = x\, t^2$,where $x = 3\, t^{1/2}$, and how to graph y vs. t in Mathematica, I could write:

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In[25]:= t = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
         x = 3 * √ (t)
         y = x * t ^ 2
         yvst = Thread[{t, y}];
         ListPlot[yvst, PlotLabel → "y vs. t", AxesLabel → {"t", "y"}]
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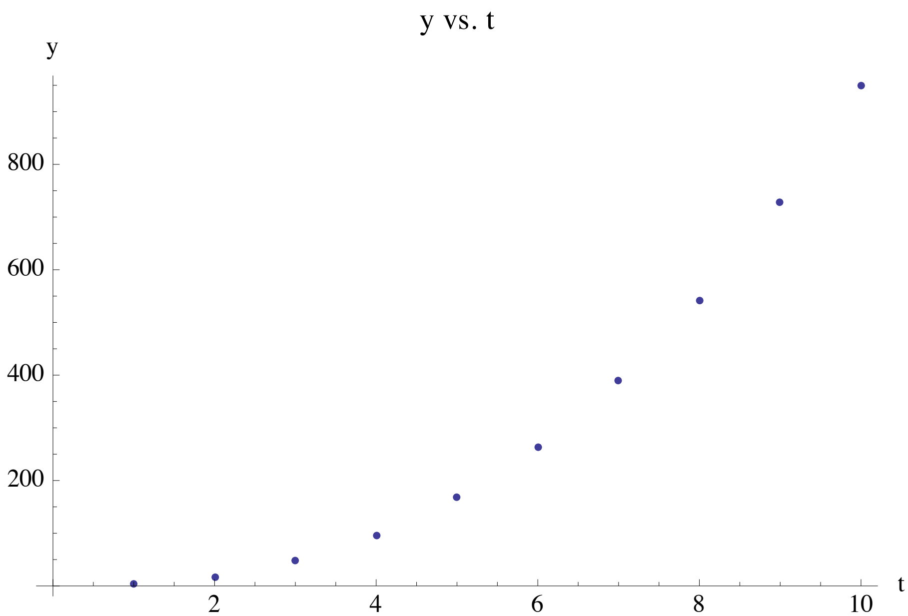
Out[26]= $\{3, 3\sqrt{2}, 3\sqrt{3}, 6, 3\sqrt{5}, 3\sqrt{6}, 3\sqrt{7}, 6\sqrt{2}, 9, 3\sqrt{10}\}$

Out[27]= $\{3, 12\sqrt{2}, 27\sqrt{3}, 96, 75\sqrt{5}, 108\sqrt{6}, 147\sqrt{7}, 384\sqrt{2}, 729, 300\sqrt{10}\}$

Out[29]=



3. What conditions must be satisfied in order to have <u>complete</u> destructive interference of two sound waves?

4. Explain with a diagram and a few words why equations (2) are the correct expressions for the two paths D_1 and D_2.

5. Sketch the graph h vs. n. [No numbers on this graph! And no calculations. Just think a minute and make a qualitative sketch, showing what the graph should look like. Ask yourself, should h increase, decrease, or stay constant as n increases.]

6. Sketch the graph λ vs. n. [No numbers! Just a qualitative sketch, showing what the graph should look like.]

7. Ultrasound is used as a tool in obstetric medicine to "see" inside the body objects larger than about a wavelength. The speed of sound in humans is about 1500 m/s, the same as in water. What would be the wavelength of 5 MHz medical ultrasound waves in humans?

8. How do you compute δv, the uncertainty in v, from measurements of f, δf, λ, and $\delta\lambda$? In this experiment, how is $\delta\lambda$ determined?

9. What is the value of $\frac{dv}{dT}$, the derivative of the speed of sound with respect to temperature, near room temperature? Sketch a graph of the speed of sound vs. temperature for temperatures near room temperature. [No numbers! Just a qualitative sketch, showing what the graph looks like.]