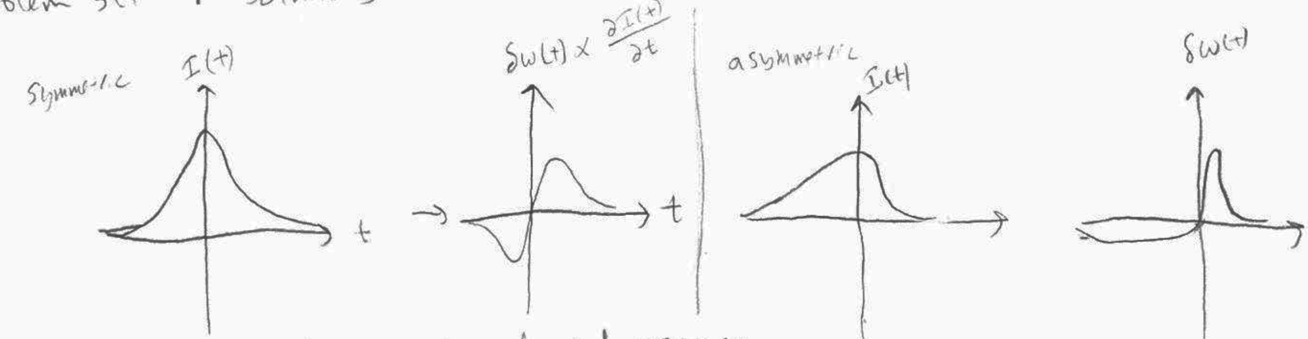
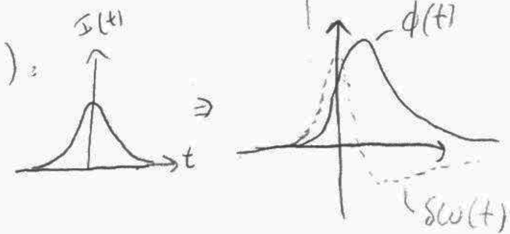
Problem set 4 - solutions

1a



b) For $\tau_r \ll \tau_0$, results approach undamped response

for $\tau_r \gg \tau_0$, asymmetry increases. For $I(t) = I_0 \operatorname{sech}^2(t/\tau_0)$:



2. Starting with non-linear wave equation, follow steps in Boyd or elsewhere to get NL Schr. Eq.

3. Plug in $E_0(z,t') = A_0 \operatorname{sech}(t/\tau_0) e^{ikz}$ into NL Schr. eq. and evaluate both sides.

You should arrive at an expression of the form

$$\gamma |A_0|^2 \operatorname{sech}^2(t/\tau_0) = K + \frac{1}{2} k_2 \frac{1}{\tau_0^2}\left(1 - 2\operatorname{sech}^2(t/\tau_0)\right)$$

For this to be true for all time t, $|A_0|^2 = -\frac{k_2}{\gamma \tau_0^2}$ and $K = -k_2/2\tau_0^2$

4. Plug the values in to $\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\varepsilon_0 \hbar} \sum_{nm} (\rho_{mm}^{(0)} - \rho_{nn}^{(0)}) \frac{\mu_{mn}^i \mu_{nm}^j}{(\omega_{nm} - \omega_p) - i\gamma_{nm}}$ where the atoms are (mostly) in the ground state $(\rho_{mm}^{(0)} - \rho_{nn}^{(0)}) \approx 1$

and use $n(\omega) = 1 + \frac{1}{2}\chi^{(1)}(\omega)$ to get $\alpha = 2\,\mathrm{Im}(n)\frac{\omega}{c} = \frac{\omega}{c}\mathrm{Im}(\chi^{(1)})$

$$\approx 10^7 - 10^8\ \mathrm{m}^{-1}$$

5. Very similar to above, using a Lorentz model for $\chi^{(1)}$ with a resonant wavelength $\sim 9\mu m$ and a similar dipole strength,

$$n(500\,\mathrm{nm}) = 1 + \frac{1}{2}\chi^{(1)}(\omega) \approx 1.5 - 2$$