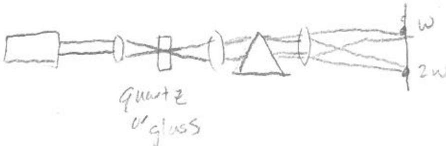
Homework 2 solutions

1. First 2nd harmonic generation

coherence length: $l_c = \frac{2}{\Delta k}$ where $\Delta k = \frac{\Delta n}{\lambda_0} = \frac{|n_\omega - n_{2\omega}|}{\lambda_0}$, so $l_c \simeq 84\,\mu m$



For an interaction volume of $10^{-11}\,cm^2$, focus size $\phi \approx 60\,\mu m$

using equation for non-depleted pump:

$$I_{2\omega} = 2 n_{2\omega} \varepsilon_0 c \left(\frac{2\omega}{n_{2\omega} c} d_{eff}\right)^2 |E_\omega|^4 z^2 \cdot \text{sinc}\left(\frac{\Delta k z}{2}\right) \simeq 1.0\,e^7\ W/cm^2$$

2. Type I: $n^o(2\omega)\cdot 2\omega = n^e(\omega,\theta)\omega + n^e(\omega,\theta)\omega$

$$n^o(2\omega) = n^e(\omega,\theta)$$

solve for θ:

$$\frac{1}{n_o(\omega)^2} = \frac{\sin^2\theta}{n_e(2\omega)^2} + \frac{\cos^2\theta}{n_o(2\omega)^2}$$

$\theta \approx 14^\circ$

Type II: $n^o(2\omega)2\omega = n^o(\omega)\omega + n^e(\omega)\omega$

Solve expression for θ.

3. Using the formulism from the paper, $\underset{\text{signal}}{\omega_3} = \underset{\text{pump}}{\omega_1} + \underset{\text{idler}}{\omega_2}$

The 3 coupled equations are

$$\frac{\partial A_3}{\partial z} = \left(\frac{2i\omega_3 d_{eff}}{n_3 c}\right) A_1 A_2 e^{i\Delta k z}$$

$$\frac{\partial A_1}{\partial z} = \left(\frac{2i\omega_1 d_{eff}}{n_1 c}\right) A_3 A_2^* e^{-i\Delta k z}$$

$$\frac{\partial A_2}{\partial z} = \left(\frac{2i\omega_2 d_{eff}}{n_2 c}\right) A_3 A_1^* e^{-i\Delta k z}$$

→ define the variables u_1, u_2, u_3, θ as

$$A_1 \equiv \left(\frac{I_{tot}}{2n_1\varepsilon_0 c}\right)^{1/2} u_1 e^{i\phi_1}$$

$$A_2 \equiv \left(\frac{I_{tot}}{2n_2\varepsilon_0 c}\right)^{1/2} u_2 e^{i\phi_2}$$

$$A_3 \equiv \left(\frac{I_{tot}}{2n_3\varepsilon_0 c}\right)^{1/2} u_3 e^{i\phi_3}$$

$$\theta \equiv \Delta k z + \phi_3(z) - \phi_2(z) - \phi_1(z)$$

→ we have 4 coupled equations.

$$\frac{du_1}{d\xi} = -u_2 u_3 \sin\theta$$

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$$\frac{du_3}{d\xi} = u_1 u_2 \sin\theta$$

$$\frac{d\theta}{d\xi} = \Delta k \cdot l - \cot(\theta)\frac{d}{d\xi}\left(\ln(u_1 u_2 u_3)\right)$$

where $\xi = z/l$, $l \equiv \frac{c}{2d_{eff}}\left(\frac{2n_1 n_2 n_3}{\varepsilon_0 c I_{tot} \omega_1 \omega_2}\right)^{1/2}$

→ using the Manley-Rowe relations, we know that

$m_1 = u_2 + u_3$, $m_2 = u_1 + u_3$, $m_3 = u_1 - u_2$

are conserved quantities, and manipulating these expressions, we can obtain an integral expression for ξ (equation 6.12 from PRB 127 1918 (1962)).

The result is of the form

$I_3(\xi) = m_2 \sin^2(m_1^{1/2}\xi)$, which is maximum when $m_1^{1/2}\xi = \frac{\pi}{2} \Rightarrow \xi = \frac{\pi}{2}\frac{1}{u_2+u_3}$

4a)

S_D - D-field poynting vector

S_E - E-field poynting vector

$$\vec{D}\cdot\vec{E} = |\vec{D}||\vec{E}|\cos\rho \Rightarrow \cos\rho = \frac{\vec{D}\cdot\vec{E}}{|\vec{D}||\vec{E}|} = \frac{D_o E_o + D_e E_e}{\left[D_o^2 + D_e^2\right]^{1/2}\left[E_o^2 + E_e^2\right]^{1/2}}$$

$$= \frac{\frac{D_o^2}{\varepsilon_0 n_o^2} + \frac{D_e^2}{\varepsilon_0 n_e^2}}{\left[D_o^2 + D_e^2\right]^{1/2}\left[\frac{D_o^2}{(\varepsilon_0 n_o^2)^2} + \frac{D_e^2}{(\varepsilon_0 n_e^2)^2}\right]^{1/2}}$$

$$\cos\rho = \frac{\frac{\cos^2\theta}{n_e^2} + \frac{\sin^2\theta}{n_o^2}}{\left[\frac{\cos^2\theta}{n_e^4} + \frac{\sin^2\theta}{n_o^4}\right]^{1/2}}$$

b) Type I: $o + o \to e$, $n_e(2\omega,\theta) = n_o(\omega)$

$$\Downarrow$$

$$\sin^2\theta = \frac{\bar{n}_e(2\omega)^2}{n_o(\omega)^2}\,\frac{n_o(2\omega)^2 - n_o(\omega)^2}{n_o(2\omega)^2 - \bar{n}_e(2\omega)^2}$$

using the expression for ρ from part (a)

we obtain

$$\tan\rho = \tfrac{1}{2} n_o^2(\omega)\left(\frac{1}{\bar{n}_e^2(2\omega)} - \frac{1}{n_o^2(2\omega)}\right)\sin 2\theta$$