Problem Set 1 – Phys 7810 – Spring 2013 Nonlinear and Nano-optics, Markus Raschke

Due date: Tu 12 February, 2013

1. We talked about the challenges with the MKS vs. Gaussian units which are amplified in nonlinear optics. But here is an easy one: Assume $\chi^{(2)} = 1 \times 10^7$ esu for some material. What is the corresponding value in MKS units considering the convention $P^2 = \epsilon_0 \chi^{(2)} E^2$.

2. For the anharmonic oscillator discussed in class, continue the perturbation expansion and derive expressions for the third-order displacement and resulting susceptibility $\chi^{(3)}$. Discuss the different terms and their resonances.

3. *Slowly varying envelope approximation*: Starting with Maxwell's equations (uniform dielectric medium, no currents, no free charges) derive the nonlinear wave equation within the slowly varying envelope approximation. Carry out the derivation which we only sketched in class in all its details. You can follow how Boyd does it, and simply using a field propagating in the z-direction only. But most importantly, sketch and discuss the meaning, and range of validity of the approximations made.

4. *Phase matched SHG*: β-barium-borate (β-BaB_2O_2, BBO) is a commonly used nonlinear optical material. A typical task you may face is to frequency double a laser beam.

   Go to the course website: http://nano-optics.colorado.edu/index.php?id=48 to download a data sheet for BBO. Familiarize yourself with the data provided. Then calculate and plot the graph describing the crystal orientation (phase matched angle) versus laser wavelength for type-I and type-II phase matched SHG.

5. *Power conversion efficiency*: We have derived the expression to describe the intensity for phase matched SHG as a function of nonlinear optical crystal parameter and laser parameter in the infinite plane wave approximation. This expression is also approximately valid for a Gaussian input beam. Consider SHG in BBO cut for phase matching with a length of 5 mm and effective nonlinear susceptibility of $\chi^{(2)} = 4.14$ pm/V. If the fundamental laser pulses have 10 nJ pulse energy, 100 fs duration, and a Gaussian beam profile with diameter of 0.2 mm, what is the energy conversion efficiency.

6. *Numerical solution to the nonlinear wave equation*: The nonlinear wave equation can be solved numerically without making any of the assumptions we made it class. Try to solve for the second-harmonic intensity as a function of crystal thickness under phase-matched or non-phase matched conditions using, e.g., BBO and for a plane wave excitation and compare with the result using the common approximations.