Problem Set 6 – Phys 4510 Optics – Fall 2014

Due date: Th, October 23, in class

Reading: Hecht 7.0, 7.1, 7.2, 7.3

1. Assume the superposition of two light waves \( E_1 = E_0 \sin(kx - \omega t) \) and \( E_2 = -E_0 \sin(kx + \omega t) \). Use the complex representation to calculate \( E(x,t) = E_1 + E_2 \). Discuss why the result is a standing wave. Derive the corresponding optical magnetic field \( B(x,t) \). Sketch both \( B(x,t) \) and \( E(x,t) \).

2. In his experiment as sketched in the figure, Otto Wiener used light at a wavelength of 589 nm provided by a Na arc lamp, spectrally filtered, and projected onto a mirror. Show that maxima in the film are observed at distances \( m\lambda/4 \), with \( m = 1, 3, 5, ... \). At what heights would be the maxima of the corresponding magnetic fields? For an angle \( \delta \) of \( 10^{-3} \) rad at what lateral intervals are the maxima observed.

Note: making the photosensitive film was a major experimental challenge in itself. It has to be significantly thinner than the wavelength to avoid producing interference itself (aside from becoming uniformly black), and it has to be largely transparent.

3. Consider a standing wave given by \( E = 2 \sin \left( \frac{2}{3} \pi x \right) \cos \left( 5\pi t \right) \). Determine the two waves superimposed to generate it.

4. Consider the superposition of three waves, one at center frequency \( \omega_c \) and two sidebands at \( \omega_c + \delta \omega \) and \( \omega_c - \delta \omega \) with your choice of frequencies and amplitudes. Use a computer program of your choice to plot the resulting wave train. Discuss your results in terms of which wave features are determined by your choice of input parameter in which way, and indicate modulation amplitude, and beat frequency.

5. An ionized gas or plasma is a dispersive medium for electromagnetic waves. To a good approximation the susceptibility is given by (which follows from the Drude model without damping) \( \chi = -ne^2/m\omega^2 \). Based on your knowledge of the relationship between \( \chi \), \( \epsilon \), and \( k \), show that the dispersion relation is given by \( k^2 = \omega_c^2 + k^2 c^2 \). What are the resulting phase and group velocities.

6. Microwave oven: conceive and execute an experiment to produce standing microwaves in a microwave oven and use it to determine its wavelength. Hint: you will need a basic model (recognizable by the turntable) that does not have a rotating antenna, sweeping frequency, or other advanced designs to avoid the generation of standing waves. What does help is try to identify the location of the antenna in your oven. As “film” drying wet paper, or spreading meltable food items, or the like.

You can do this experiment in groups, but everybody shall submit their own brief report.