Problem Set 4 – Phys 4510 Optics – Fall 2014

Due date: Th, October 2, in class

Reading: Hecht 4.7 - 4.8

1. Prism: we have already discussed that the prism is a very universal optical device. In this problem we would like to study its diffractive effects. Consider the equilateral prism shown in Fig. 1 with apex angle α, and made of a materials of index of refraction n.

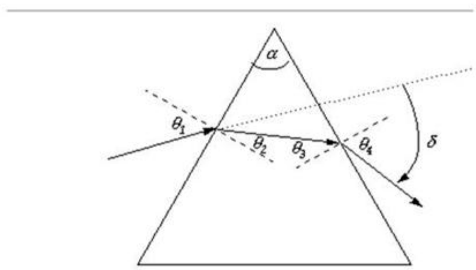

Figure 1:

   a) Derive an expression for the deflection δ as a function of θ_1. Note: it is clear that you can find that solution in many books, including Hecht, but I encourage you to do it yourself, and only use other resources to check your results. Hint: if you copy blindly from a solution you find, you may score well on the homework, but poorly on the exam.

   b) For a prism with $n = 2.5$ and an angle α of your choice, plot $\delta(\theta_1)$ using a computer program of your choice. Optional: for the enthusiasts, make a 3D plot $\delta(\theta_1, \alpha)$.

   c) Assume you enter under an angle such that $\theta_2 = \theta_3$, i.e., the beam propagates inside the prism parallel to its base. Would the beam exit the prism on the right side? If not, would it exist somewhere else? How does the situation depend on α?

   d) Now we consider the frequency dependence of n. Assume $n = 2.4$ for red and $n = 2.6$ for blue light, and $\alpha = 40°$. What is the angular spread $\Delta\delta$ for the two colors?

2. Evanescent wave: In class we were shining a green laser pointed ($\lambda = 532$ nm, $n = 1.5$) under variable internal angles onto the base of the prism to demonstrate the effect of total internal reflection. At what distance from the surface is the amplitude of the evanescent wave $1/e$ of its value at the surface? By what factor is the intensity of the evanescent wave reduced at a distance of 0.1 mm from the surface?

3. Optical fibers: optical fibers have gained large technological importance for data transfer over long distances. Let's study some key design criteria relevant for their performance.

   a) Show that the acceptance angle for a glass fiber is given by $\theta_i = sin^{-1}\sqrt{n_1^2 - n_2^2}$ where n_1 and n_2 are the indices of refraction of the fiber and the cladding, respectively. Typical fibers are made of a quartz core ($n_1 = 1.46$) and a cladding glass with slightly lower index ($\delta n \simeq 0.02$). What is the angle of total internal reflection? What is the value of the acceptance angle?

   b) For long distances there is unavoidable loss due to absorption and scattering. This necessitates amplifiers at regular distances. Consider again a quartz fiber, now at a specific wavelength of 1550 nm, with $\tilde{n} = n + ik$ with $n = 1.5255$ and $k \approx 1 \times 10^{-7}$. Calculate the absorption coefficient and the damping D (in dB/m). How large is the intensity decay after a distance of 1 km.

c) In order to minimize loss, one resorts to ultra-pure quartz glass with a damping of D = 0.15 dB/km. If you implement an amplifier every 100 km, what amplification do you required to restore the signal to its original value, i.e., the ability to transmit the signal over arbitrary distances?

4. Rainbow:

   a) The formation of a rainbow is the result of refraction, reflection, and dispersion in droplets of water in air. The full details of the process are rather complicated, but the following situation allows you to qualitatively understand the phenomenon. Consider a droplet of water and parallel light rays incident at a distance b parallel to an axis (dotted line). We are interested in the portion of light that is refracted into the sphere, reflected at the other side, and then again refracted when exiting the sphere. Calculate the angle θ as a function of the parameter b for a drop of radius r, and refractive index $n = 1.33$ (for $\lambda = 589$ nm). Plot the results using a program of your choice.

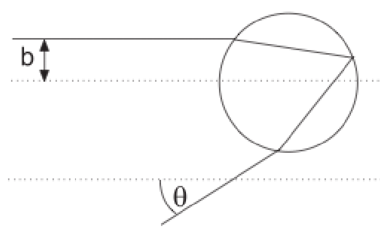

Figure 2:

   b) The index of refraction is wavelength dependent $n(\lambda)$. For water in the optical spectral range, the empirical relationship $n = 1.33 + 0.012\frac{706-\lambda}{300}$ provides a good description with λ in nm. Calculate the divergence of the rainbow in the visible spectral range. Can one learn something about the drop size from observing a rainbow?

   c) Why does one sometimes observe two rainbows?

5. The Drude model despite its simplicity describes surprisingly well the frequency dependence of the dielectric function $\epsilon(\omega)$ for metals up to the visible spectral region.

   Here, we want to take a closer look at Au and Ag characterized by the following parameter:

   Au: $n = 5.9 \times 10^{28}$ m^{-3}, $\sigma = 4.9 \times 10^7$ Ωm^{-1}, $m^*/m_e = 0.99$
   Ag: $n = 5.76 \times 10^{28}$ m^{-3}, $\sigma = 6.6 \times 10^7$ Ωm^{-1}, $m^*/m_e = 0.96$

   a) From the DC conductivity derive the electron relaxation time (in fs) or in other words the time between collisions with other electrons, lattice ions or defects.

   b) Calculate the values for the plasma frequency for Au and Ag (give values in eV).

   c) Plot the Drude dielectric function Re(ϵ) and Im(ϵ) for Au and Ag using (i) $\epsilon_\infty(Au) = 9.84$, and (ii) $\epsilon_\infty(Ag) = 3.7$. A useful energy range to look at is from 0.5 to 3.5 eV.

   d) Plot the skin depth for the two cases.

   e) Now we pick Au at a wavelength of 800 nm: how thick does a Au film have to be to attenuate the light intensity by 50%. How many atomic layers does this correspond to.

   For your information: while the model works very well for Ag, in the case of Au for energies above about 1.8 eV, especially Im(ϵ) starts to deviate due to interband contributions.