Problem Set 1 – Phys 4510 Optics – Fall 2014

Due date: Tu, September 9, in class

Reading: Hecht, skim Ch. 2, thoroughly read Ch 3 introduction, 3.1, 3.2, 3.3.1, 3.3.2, 3.4.1, 3.4.3.

1. Tell me in a few words about your main interests in physics, your current or possible plan to do research in optics, your plan to go to graduate school or industry or other after graduating, etc. This will help me fine tune topics we cover.

2. We derived the wave-equation for the electric field $E(y,t)$ which reads for the 1D case:

$$\frac{\partial^2 E(y,t)}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E(y,t)}{\partial t^2} = 0 \tag{1}$$

   a) Show that the ansatz $E(y,t) = E_0 cos(\omega t - ky + \phi)$ is a solution of the wave equation, with wavevector k, angular frequency ω, and phase ϕ. What are the units of k, ω, ϕ, and c? What is the relationship between k and ω (it is called dispersion relation)?

   b) What is the spatial and temporal evolution of the plane wave? Discuss, whether and how real physical waves can be represented by plane waves.

   c) Sketch the wave at $t = 0$ and $t = t'$ for $0 < t' < 2\pi/\omega$. What can you say about the propagation direction of the wave? What is the velocity with which the crest of the wave propagates along the y-direction?

3. Consider a electromagnetic plane wave propagating in the x-direction, and polarized in the y-direction.

   a) Show from Maxwell's equations that

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \tag{2}$$

   b) Let us now assume a specific harmonic wave of the form $E_y(x,t) = E_{y,0} cos\{\omega(t - x/c)\}$. What is the corresponding expression for the magnetic field $B_z(x,t)$? Also show that $E_y = cB_z$.

   c) Sketch in a single 3D graph E_y and B_z as they propagate in the x-direction. What is the phase relationship between E_y and B_z? Lastly calculate the corresponding Poynting vector (component) S_x.

4. This problem shall give you a feeling for intensities, field amplitudes, and frequencies of light in the visible and infrared. Consider two cw laser sources, (1) a green, e.g., HeNe laser ($\lambda = 543$ nm), and (2) an IR, e.g., CO_2 laser ($\lambda \sim 10\mu$m), both with a power of 10 mW each.

   a) Calculate for each laser light in vacuum the frequency, the wavevector, the photon energy (in eV, and in multiples of kT at room temperature), and the intensity in number of photons per second. Note on units: quantum energy is most conveniently given in eV, wavevector in cm^{-1}).

   b) Assume you focus the light using appropriate optics down to a spot size of 10μm diameter. What is the irradiance (aka intensity) [most convenient in W/cm^2]? What is the electric field amplitude? What is the corresponding magnetic field amplitude?

   d) Now we use a 100 W light bulb converting say 2% of its electric power into light. Distribute the radiation evenly over a solid angle $\Omega = 1$ sterad by a reflector. Calculate the light intensity and electric field of the light field at a distance of 1m from the bulb.

   c) Discuss how intensity and field strength compare if you now were to use sunlight collected with a 5 cm diameter lens, still focused to a spot size of 10μm diameter (For this problem start from your knowledge of the luminosity of the sun of $L = 3.9 \times 10^{26}$ W, and the distance of the earth from the sun of 1.5×10^8 km; neglect atmospheric effects).