Physics 1140 Fall 2012

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Lecture #4:

1. Misc. things from the lab
2. Statistical analysis (part 1)

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Fresh from the lab…

1. Unhappy? Talk to us!
2. Lab reports can be written at home
3) With a math program of your choice (some restrictions apply).

Lab reports:

4. Compar your results with theory.
5. Discuss possible origin of deviations.
6. Units, labels, errors, significant figures,…

Where do we stand …

1. Many of you are still struggling with value and error in “standard format”! See lecture 2 for details.
2. Do the hw. It may throw your grade off more than you may think, as you will likely not understand how to apply the concepts in the lab reports if you do not practice.

Didn't turn in HW.	
9.866	0.1007 !!!
9.79	0.01
9.831	0.005
9.82	0.05
Didn't do problem 4.	
Didn't turn in HW.	
9.799	0.017
9.745	0.006
9.82	0.02
9.811	0.006
9.6504	0.01 !!!
Didn't turn in HW.	
9.726	0.005
Didn't turn in HW.	
Didn't turn in HW.	
9.796	0.041 !!!
9.838	0.03 !!!
9.86	0.006 !!!
9.801	0.016

Table is result from one section.

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Recap:

- General rule of error propagation:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + ...}$$

- Example pendulum:

$$g = 4\pi^2 \frac{L}{T^2} \quad ; \quad \delta g = \sqrt{\left(\frac{\partial g}{\partial L}\delta L\right)^2 + \left(\frac{\partial g}{\partial T}\delta T\right)^2}$$

$$\frac{\partial g}{\partial L} = 4\pi^2 / T^2 \quad ; \quad \frac{\partial g}{\partial T} = -2(4\pi^2 L / T^3)$$

$$\delta g = \sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3}\delta T\right)^2}$$

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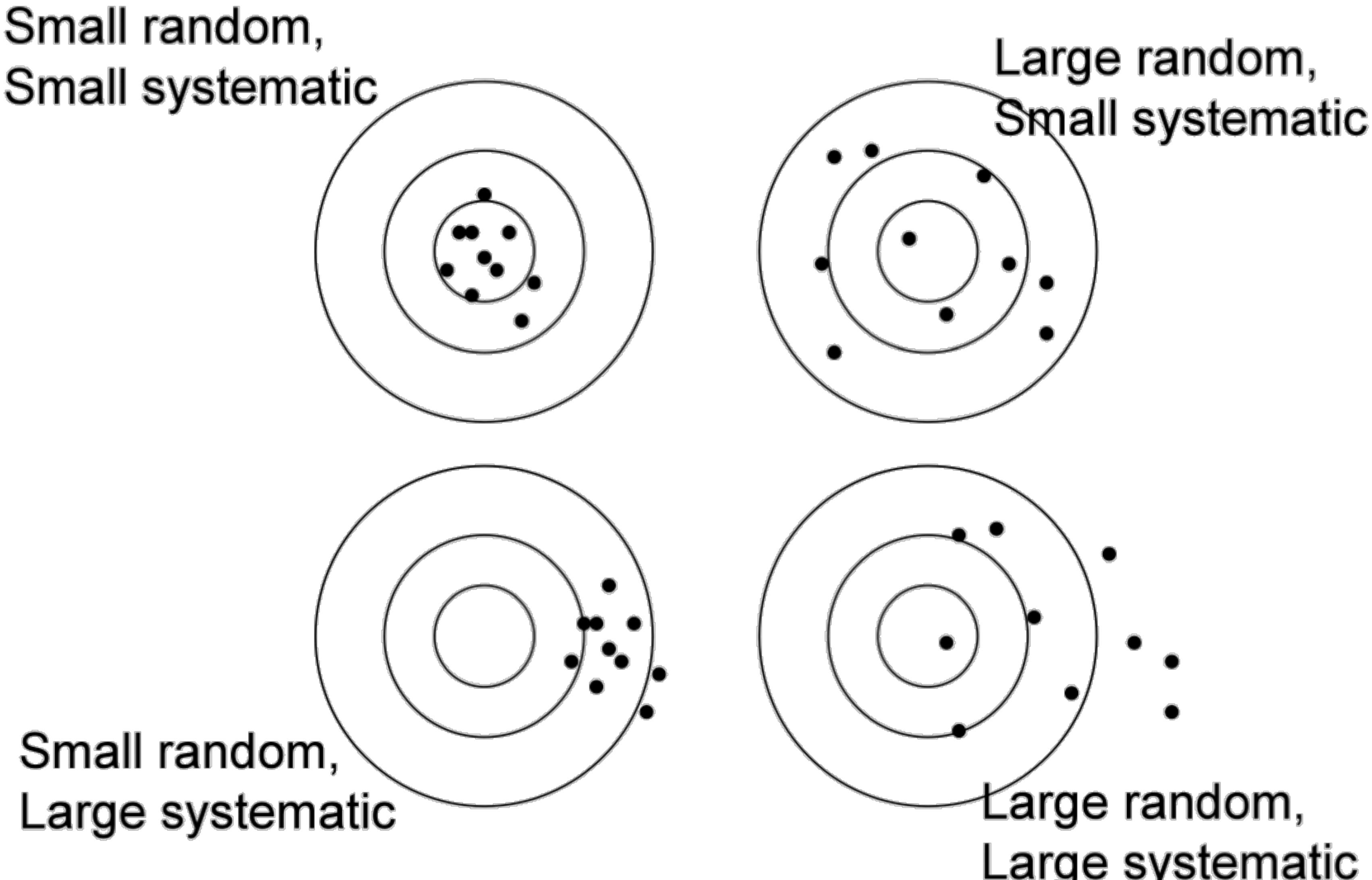
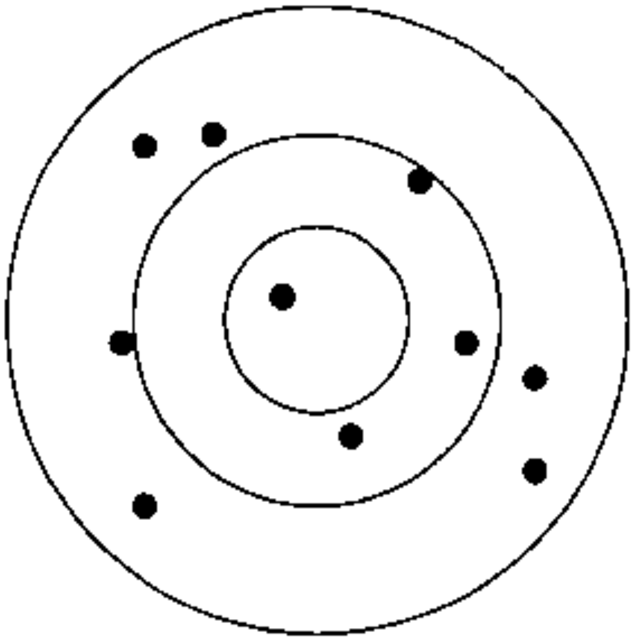
Characterization of errors



- Large *random* error



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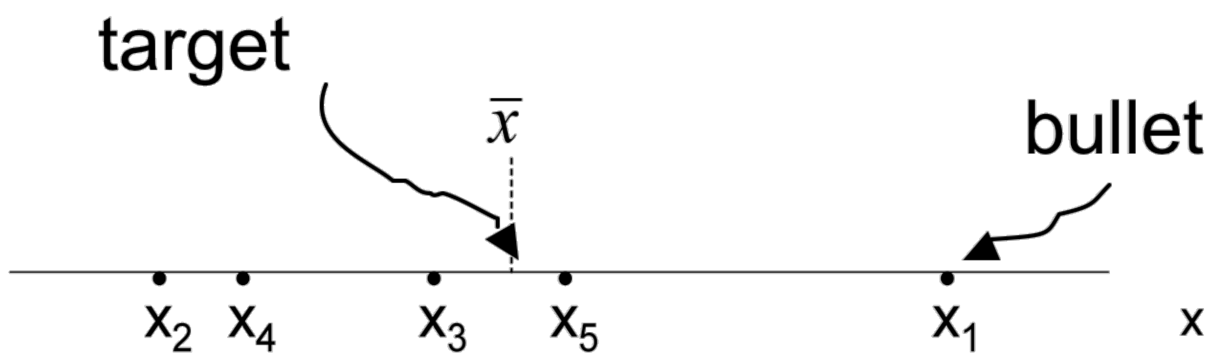
How do we distinguish?

- Random errors
  - Try try again…..
  - Make a bunch of measurements of x, average



$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{x} = x_{avg} = x_{mean} = \langle x \rangle$$

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Statistical averaging

- When there is only random error
  - *NO* systematic error

- then in the limit of an infinite number of measurements, the average is the “target”

- Even if you only take a few “shots” (5 is a good number), the average is usually closer to the target than most of the typical individual shots.

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Example

- Make 6 measurements of x
  - x = 2, -2, -1, 2, 5, 0

$$\langle x \rangle = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{6}(2-2-1+2+5+0) = \frac{1}{6}6 = 1$$

$$\bar{x} = 1$$

  - In this example, if the average value is 1, then
    - Measured values vary from -2 = (1-3) to 5 = (1+4)
    - Reasonable estimate of error is 1 +/- 3 or 1 +/- 4

Standard deviation

- To have a more precise estimate of the error in any particular measurement, we could calculate the *average deviation* of our data from the average:

$$\overline{\left(x_i - \bar{x}\right)}$$

  – Well, *this* value turns out to be zero – as many negative as positive

    - So let's do:

$$\overline{\left(x_i - \bar{x}\right)^2}$$

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Standard deviation and variance

- The definition of *standard deviation* is:

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

  - *Note that this includes N-1, rather than N, in the denominator.*

- There is also a name for this w/o the square root
  - The *variance:*

$$\sigma_x{}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

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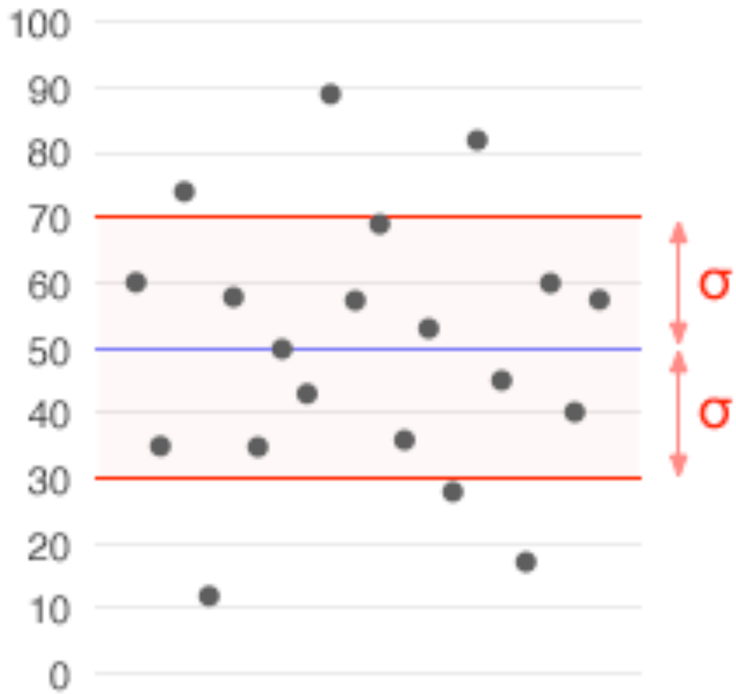
Standard deviation as random error



$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

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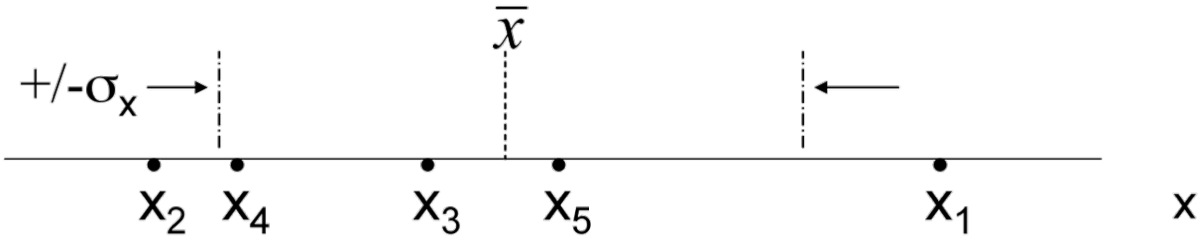
Standard deviation as random error

- Mean and standard deviation:

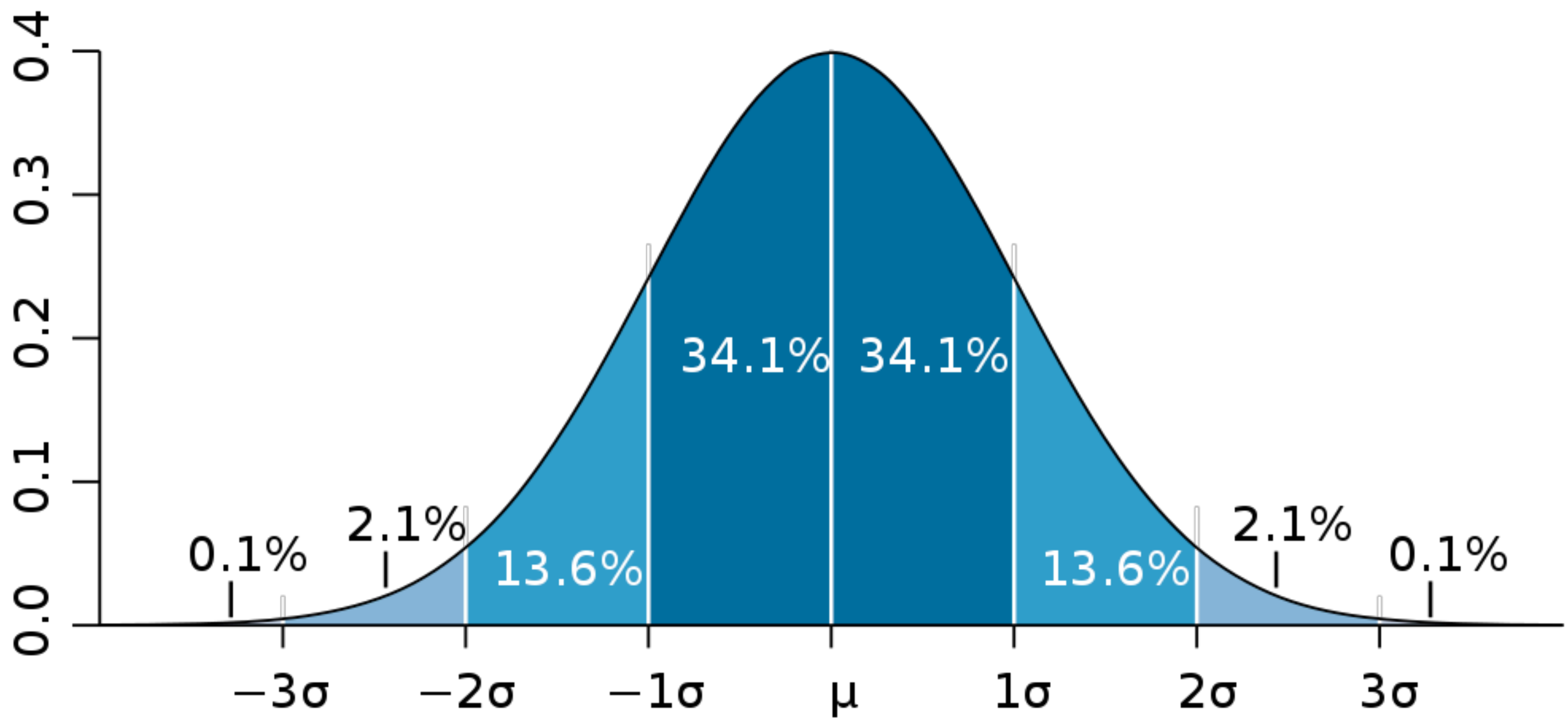


  - If we make *a single measurement*, the actual value is probably within one *standard deviation* σ_x of our measurement,

$$x \pm \delta x = \bar{x} \pm \sigma_x \;\; ; \;\; \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \;\; ; \;\; \sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

The normal distribution

- 68% of the time the measurement is within σ
  - 2x(34.1+13.6)~95% of the time within 2σ
  - 99.7% of the time within 3σ



Back to our example

- Make 6 measurements of x
  - x = 2, -2, -1, 2, 5, 0 $\qquad \bar{x} = 1$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} =$$

$$= \sqrt{\frac{1}{N-1}\left((2-\bar{x})^2 + (-2-\bar{x})^2 + (-1-\bar{x})^2 + (2-\bar{x})^2 + (5-\bar{x})^2 + (0-\bar{x})^2\right)}$$

$$= \sqrt{\frac{1}{N-1}\left((1)^2 + (-3)^2 + (-2)^2 + (1)^2 + (4)^2 + (-1)^2\right)}$$

$$= \sqrt{\frac{1}{N-1}\left(1+9+4+1+16+1\right)} = \sqrt{\frac{1}{N-1}\left(32\right)}$$

$$= \sqrt{6.4} = 2.5$$

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What does this actually mean? Not trivial…..

- Make 6 measurements of x
  - x = 2, -2, -1, 2, 5, 0
  - $\bar{x} = 1$
  - $\sigma_x = 2.5$

- If we make a *single* measurement,
  - *That number* is likely to be within ± 2.5 of the actual, correct value.

- On the other hand, if we keep measuring, and keep averaging, $\bar{x} = 1$ should be *more* accurate than within ± 2.5.

Apply our rule of error propagation:

- For our 6 values, we have

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{6}(2-2-1+2+5+0) = \frac{1}{6}6 = 1$$

$$\bar{x} = 1$$

- *Each* of these values has an error of +/- dx=2.5
  - *For our six values:*

$$\bar{x} = f(x_1, x_2, x_3, x_4, x_5, x_6) = \frac{1}{6}\left(x_1 + x_2 + x_3 + x_4 + x_5 + x_6\right)$$

  - *And the error is now:*

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\delta x_2\right)^2 + \left(\frac{\partial f}{\partial x_3}\delta x_3\right)^2 + \dots}$$

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- Turn the crank:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\delta x_2\right)^2 + \left(\frac{\partial f}{\partial x_3}\delta x_3\right)^2 + \ldots.}$$

$$f(x_i) = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \;\; ; \;\; \frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial x_3} \ldots = \frac{1}{N}$$

$$\delta x_1 = \delta x_2 = \delta x_3 \ldots = \sigma_x$$

$$\delta f = \delta\bar{x} = \sqrt{\left(\frac{1}{N}\sigma_x\right)^2 + \left(\frac{1}{N}\sigma_x\right)^2} + \ldots. = \sqrt{N\left(\frac{1}{N}\sigma_x\right)^2}$$

$$\delta f = \delta\bar{x} = \sqrt{\frac{\sigma_x{}^2}{N}} = \frac{\sigma_x}{\sqrt{N}}$$

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The result

- Repeated measurements of the same parameter, when *subject to random error only,* and when averaged, home-in on the "actual" value with progressively smaller error:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- With a final result $\bar{x} \pm \sigma_x / \sqrt{N}$
  - This error $\sigma_x / \sqrt{N}$ is called the standard error on the mean