# *Physics 1140 Fall 2012*

Prof. Markus B. Raschke

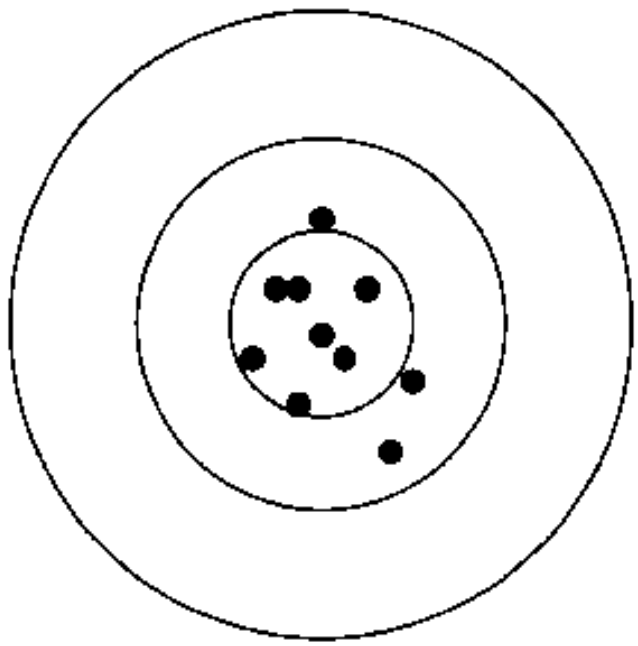
Lecture #3:

1. Error propagation in calculations

Colorado
University of Colorado at Boulder

## *Making measurements in the lab*

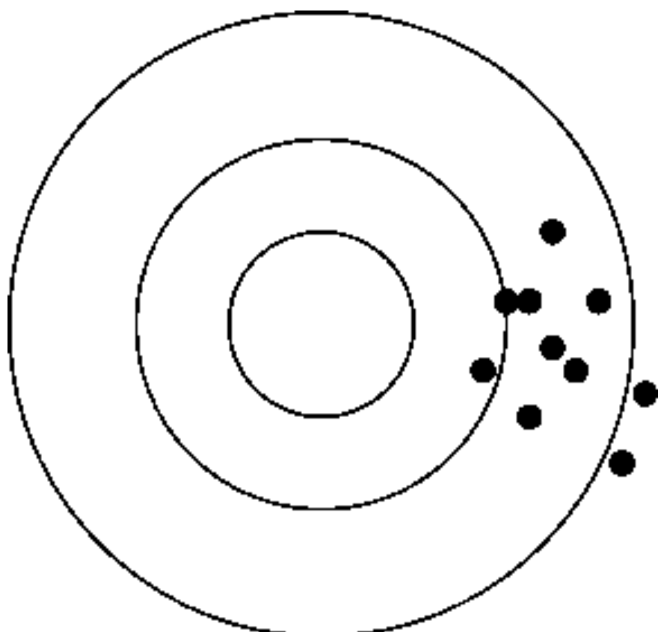
- You're always trying to home-in on the right answer.
  - Like target practice
    - If at first you don't succeed, try again…..
  - But really, in science we generally don't know *where* the bullseye is:



Colorado
University of Colorado at Boulder

## *Making measurements in the lab*

- You're always trying to home-in on the right answer.
  - Like target practice
  - But really, in science we generally don't know *where* the bullseye is, and we might be doing the measurement wrong:

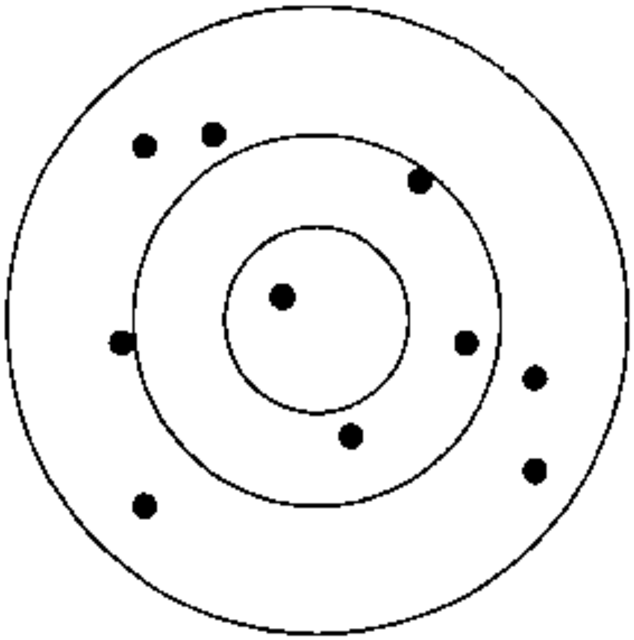


  - *Systematic* error

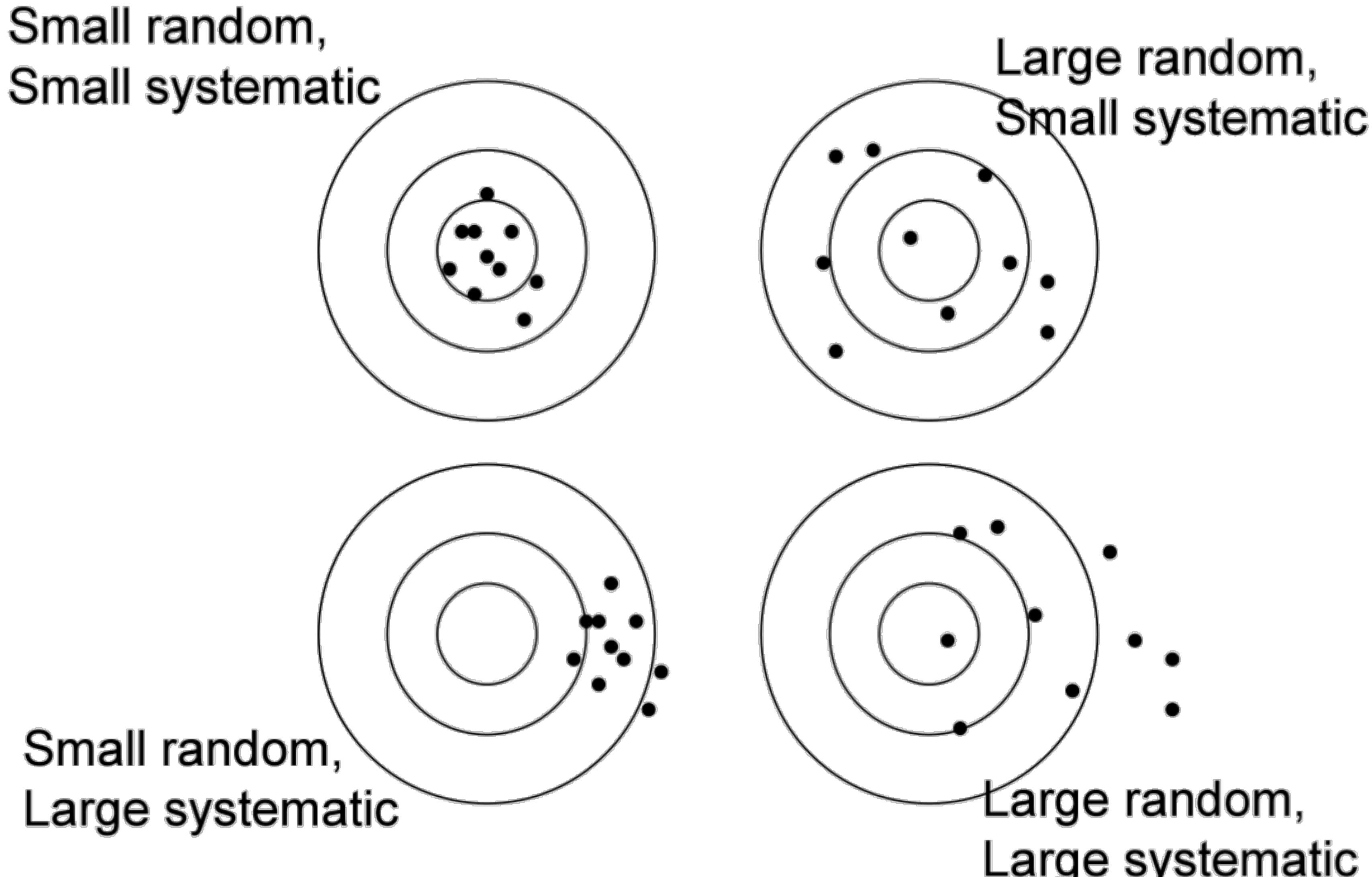
Colorado
University of Colorado at Boulder

## *Or we might not be such a great shot:*

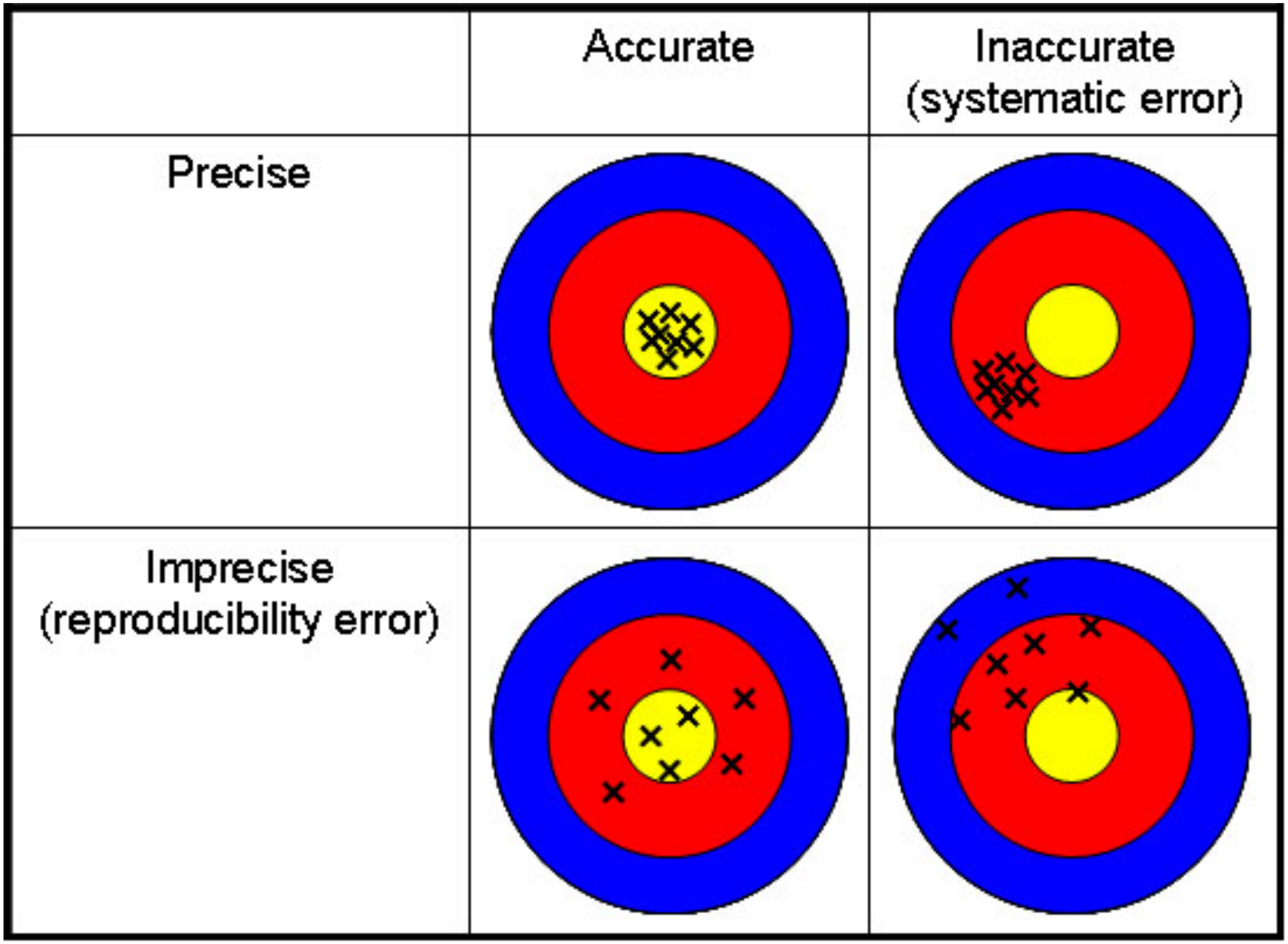
- Large *random* error



# *Characterization of errors*



# *Accuracy vs. Precision*



# *Propagating error estimates in calculations*

- Addition and subtraction
  - Step 1: Do the math
  - Step 2: **new error is square root of the sum of the squares** of the individual errors

- A+B with A±dA;  B±dB:
  - result is

$$(A+B)\pm\sqrt{\delta A^2+\delta B^2}$$

  - A-B:
    - Result is

$$(A-B)\pm\sqrt{\delta A^2+\delta B^2}$$

Colorado
University of Colorado at Boulder

## *Propagating error estimates in calculations*

- Example
  - Measure A= 23.6 ± 0.2 cm ; B= 262.5 ± 0.5 cm
  - Sum: (A+B) = 286.1 cm
  - Error
    - First, if the error in B is ± 0.5 cm, the total error *can't* be less
    - But simply adding 0.5+0.2=0.7 is generally too pessimistic (why?)
    - Calculate:

$$\sqrt{0.5^2 + 0.2^2} = \sqrt{0.25 + 0.04} = \sqrt{0.29} = 0.58 \approx 0.6$$

- Important:
  - This rule applies only for *uncorrelated* (random) errors

Colorado
University of Colorado at Boulder

## *Another example: fractional errors*

- Velocity V = X/T
  - X = 100 +/- 5 m (5% fractional error; i.e. dX/X=0.05)
  - T = 10.0 +/- 0.5 s (5% fractional error also)
  - dX and dT are uncorrelated

- dV/V is?

$$\delta V / V = \sqrt{\left(\delta X / X\right)^2 + \left(\delta T / T\right)^2}$$

$$= \sqrt{0.05^2 + 0.05^2} = \sqrt{2 \cdot 0.05^2} = \sqrt{2} \cdot 0.05 = 0.07$$

Colorado
University of Colorado at Boulder

## *Exact numbers in formulas*

- Exact numbers have zero error
- If C=BA
  - A=2.34 +/- 0.03 sec
  - B=2 (exact)   +/- 0

$$\delta C / C = \sqrt{\left(\delta A / A\right)^2 + \left(\delta B / B\right)^2}$$

$$\delta C / C = \sqrt{\left(\delta A / A\right)^2} = \delta A / A \qquad \delta C = C \bullet \delta A / A = 2A \bullet \delta A / A = 2\delta A$$

  - C=4.68 +/- 0.06 sec

## *Master rule of error propagation:*

- Given *uncorrelated* uncertainties dx, dy, dz,….. And a function f(x,y,z….) the uncertainty in f is:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + .....}$$

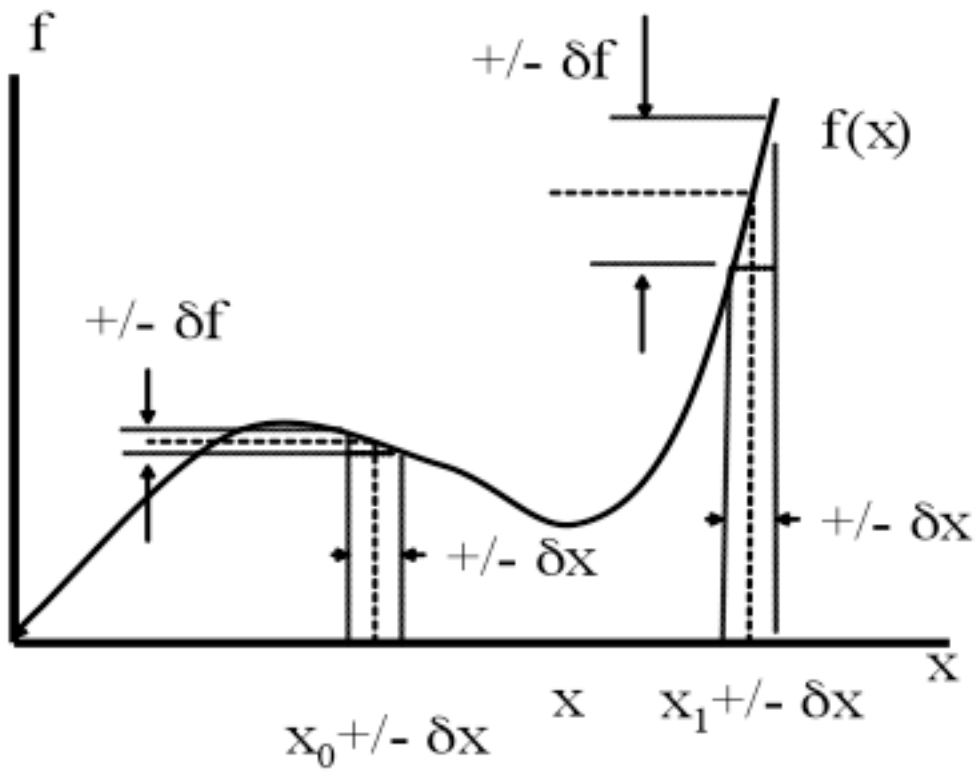
This might well be the single most important equation of this class.

Colorado
University of Colorado at Boulder

# *Graphically*

$x \; +/- \; \delta x$

$\delta f = ?$




The general rule: $\delta f = \left| \delta x \dfrac{df}{dx} \right|$

Colorado
University of Colorado at Boulder

## *Example: Simple Pendulum*

- Period of pendulum T is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

  - where **L** is length of pendulum
  - **g** is the acceleration of gravity.

- Measure **L ± dL**, and **T ± dT**. What is d**g?**

- First, rewrite it: $$g = 4\pi^2 \frac{L}{T^2}$$

## *Turn the crank:*

- Start:

$$\delta g=\sqrt{\left(\frac{\partial g}{\partial L}\delta L\right)^2+\left(\frac{\partial g}{\partial T}\delta T\right)^2} \quad ; \qquad g=4\pi^2\frac{L}{T^2}$$

- So that:

$$\frac{\partial g}{\partial L}=4\pi^2/T^2 \quad ; \quad \frac{\partial g}{\partial T}=-2(4\pi^2 L/T^3)$$

$$\delta g=\sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2+\left(\frac{8\pi^2 L}{T^3}\delta T\right)^2}$$

- And the *fractional* error is:

$$\frac{\delta g}{g}=\frac{1}{g}\sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2+\left(\frac{8\pi^2 L}{T^3}\delta T\right)^2} \quad ; \quad \frac{1}{g}=T^2/(4\pi^2 L)$$

$$\frac{\delta g}{g}=\sqrt{\left(\frac{\delta L}{L}\right)^2+\left(\frac{2\delta T}{T}\right)^2}$$

- **As given in the lab manual!**

Colorado
University of Colorado at Boulder

## *Weird functions:*

- Don't panic– just apply the general rule!
  - For instance, $f(x,y)=\sin(xe^{ay})$
  - !!!!!!!!!!!!!!!

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots\ldots}$$

$$\frac{\partial f}{\partial x} = \cos(xe^{ay})e^{ay} \quad , \quad \frac{\partial f}{\partial y} = \cos(xe^{ay})axe^{ay}$$

plug in your partial derivatives .....

$$\partial \mathrm{f} = \sqrt{(\cos(xe^{ay}\delta x)e^{ay})^2 + (\cos(xe^{ay})axe^{ay}\delta y)^2}$$

..... and you're done!

Colorado
University of Colorado at Boulder