Physics 1140 Fall 2011

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Lecture #3:

1. Error propagation in calculations (ch 3)

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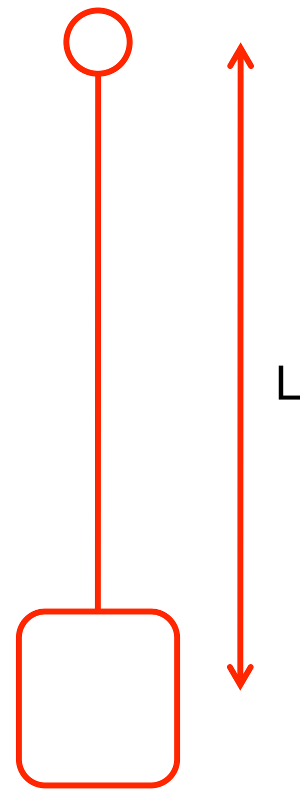
Fresh from the lab…

1. **Chaos rules**.
2. NEW: lab reports can be written at home (see email).
3. Lab reports:
4. Discuss your errors.
5. Comparison with theory.
6. Units, labels, errors, significant figures,…

New homework:

g revisited:

$$g = 4\pi^2 \frac{L}{T^2}$$



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Lyrics and Music by Princess Clemente

http://www.youtube.com/watch?v=EVSS9FXSaI0

Gravity has disappeared; you've swept me off my feet.
You're not just any normal force that makes my heart beat
Cuz everytime I see your face, my heart accelerates.

Don't mean to go on a tangent but baby whats your sine?
Can I have your digits? Can I please make you mine?
I promise I can be your significant figure.

I'm falling 9.8 meters per second squared for you.
The center of mass is my heart, heavy in love with you.

Don't leave me hanging like a Foucault pendulum.
I've got the potential energy to make some kinetic fun.
Inelastic collision! Turn our hearts into one!

Your love is at my own fundamental frequency.
Our circuit of love has no resistors, you see
But if tensions do grow, we'll learn to find our constant.
Just think absolute value, everything will turn out positive.

I'm falling 9.8 meters per second squared for you.
The center of mass is my heart, heavy in love with you.

Gravitational attraction brings me closer to you.
If I can only derive the formula to make you love me too.
I hope you like the value of mu--- sic.

I'm falling 9.8 meters per second squared for you.
The center of mass is my heart, heavy in love with you.

Lets get physical!

I'm falling 9.8 meters per second squared for you.

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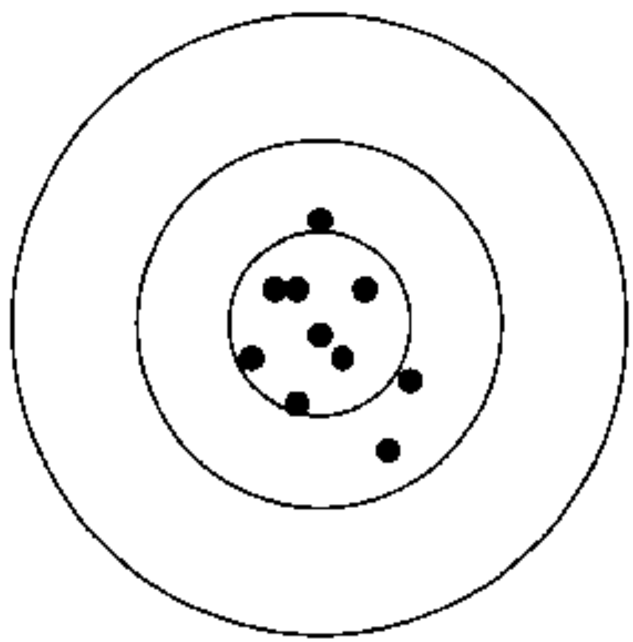
Making measurements in the lab

- You're always trying to home-in on the right answer.
  - Like target practice
    - If at first you don't succeed, try again…..
  - But really, in science we generally don't know *where* the bullseye is:



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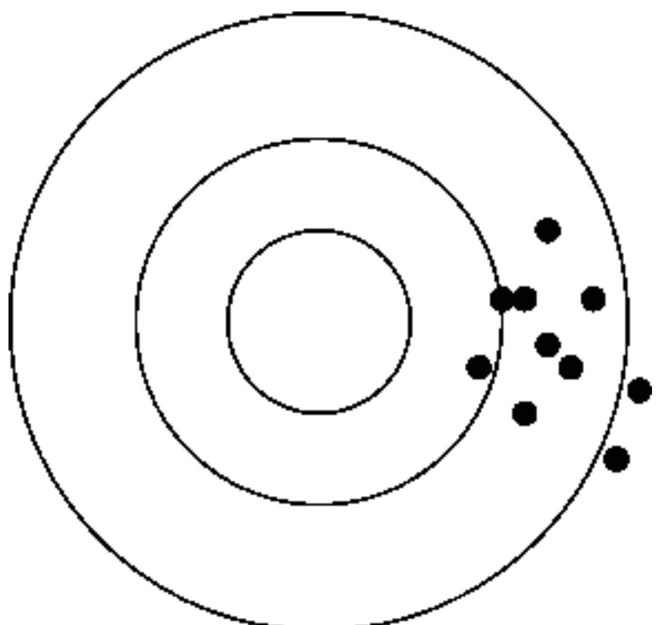
Making measurements in the lab

- You're always trying to home-in on the right answer.
  - Like target practice
  - But really, in science we generally don't know *where* the bullseye is, and we might be doing the measurement wrong:



  - *Systematic* error

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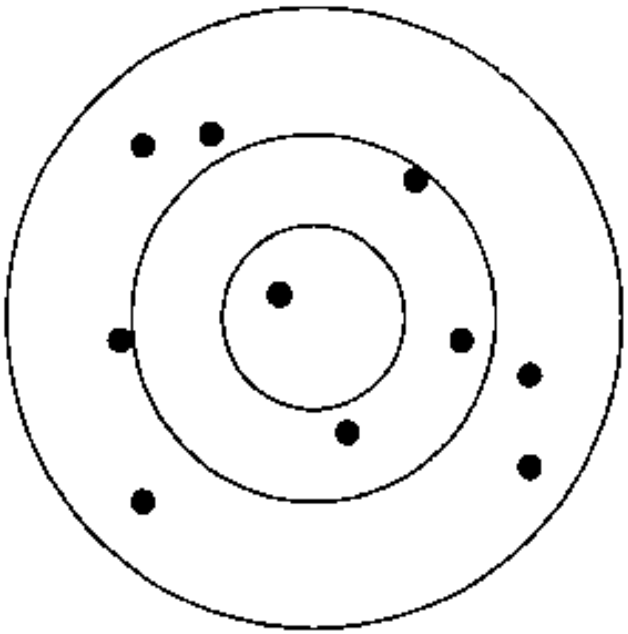
Or we might not be such a great shot:

- Large *random* error



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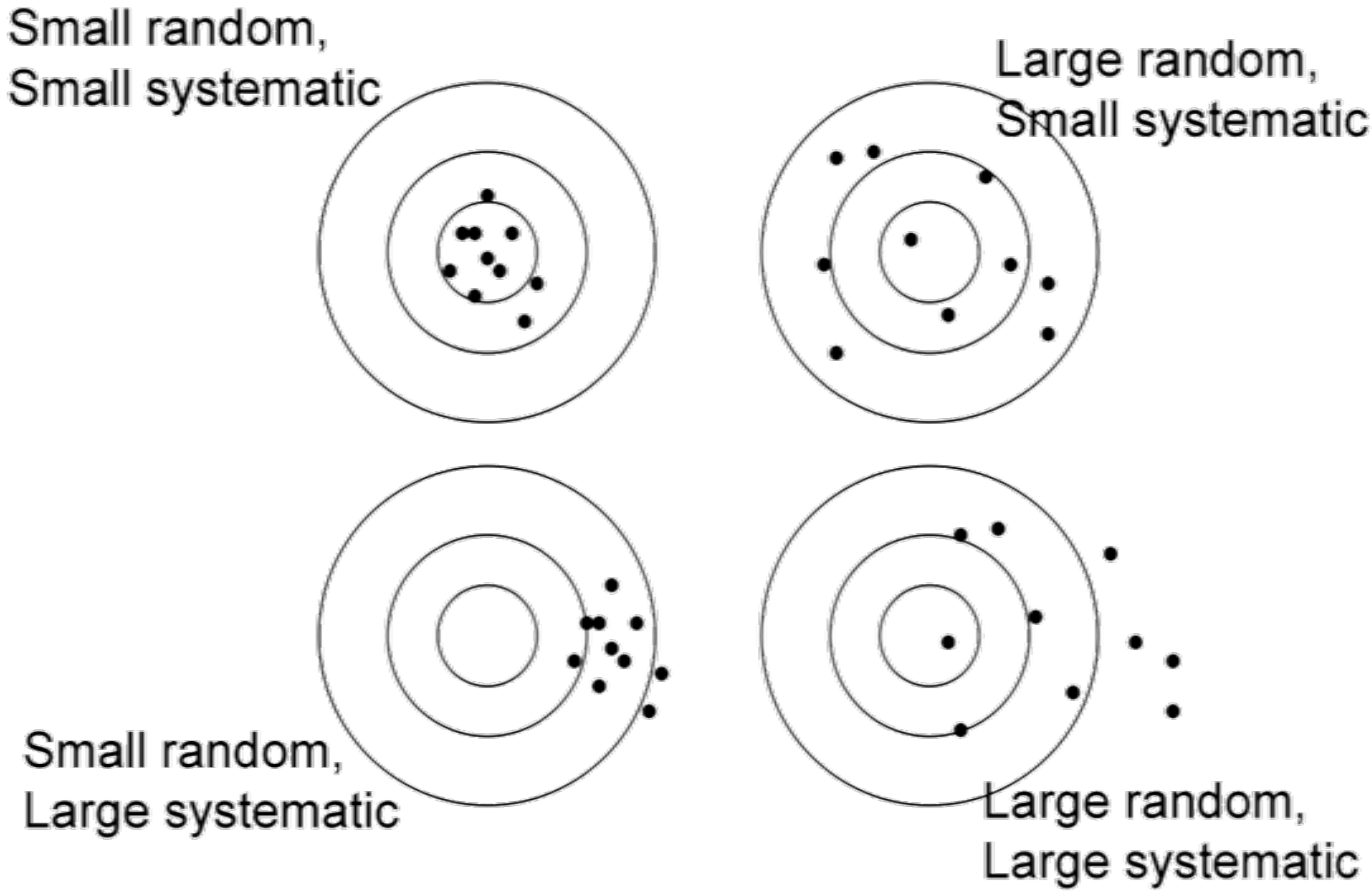
Characterization of errors



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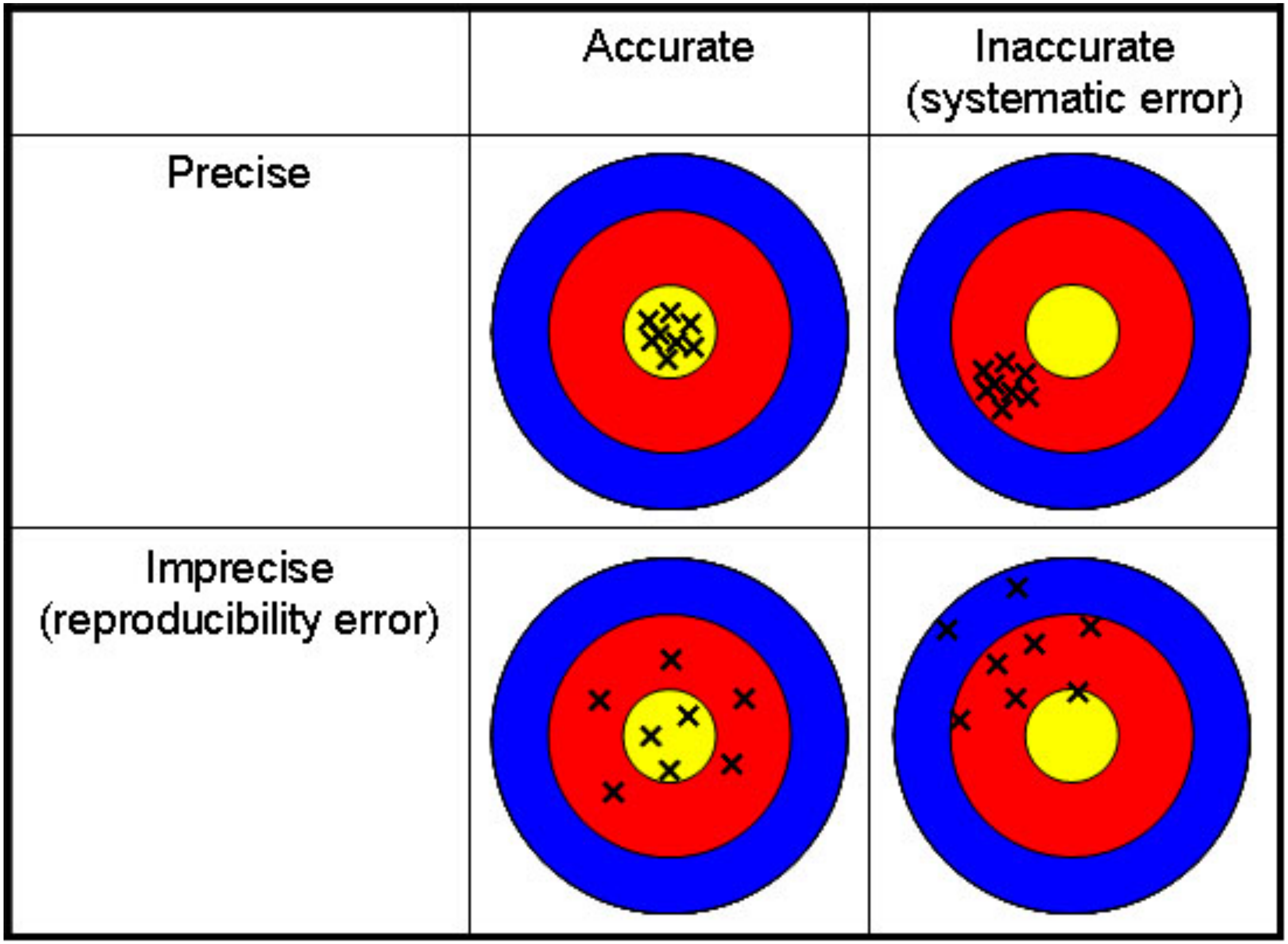
Accuracy vs. Precision



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Master rule of error propagation:

- Given *uncorrelated* uncertainties dx, dy, dz,…..
  And a function f(x,y,z….) the uncertainty in f is:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots\ldots}$$

This might well be the single most important equation of this class.

Propagating error estimates in calculations

- Addition and subtraction
  - Step 1: Do the math
  - Step 2: **new error is square root of the sum of the squares** of the individual errors

- A+B with A±dA; B±dB:
    - result is

$$(A+B) \pm \sqrt{\delta A^2 + \delta B^2}$$

  - A-B:
    - Result is

$$(A-B) \pm \sqrt{\delta A^2 + \delta B^2}$$

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Propagating error estimates in calculations

- Example
  - Measure A= 23.6 ± 0.2 cm ; B= 262.5 ± 0.5 cm
  - Sum: (A+B) = 286.1 cm
  - Error
    - First, if the error in B is ± 0.5 cm, the total error *can't* be less
    - But simply adding 0.5+0.2=0.7 is generally too pessimistic (why?)
    - Calculate:

$$\sqrt{0.5^2 + 0.2^2} = \sqrt{0.25 + 0.04} = \sqrt{0.29} = 0.58 \approx 0.6$$

- Important:
  - This rule applies only for *uncorrelated* (random) errors

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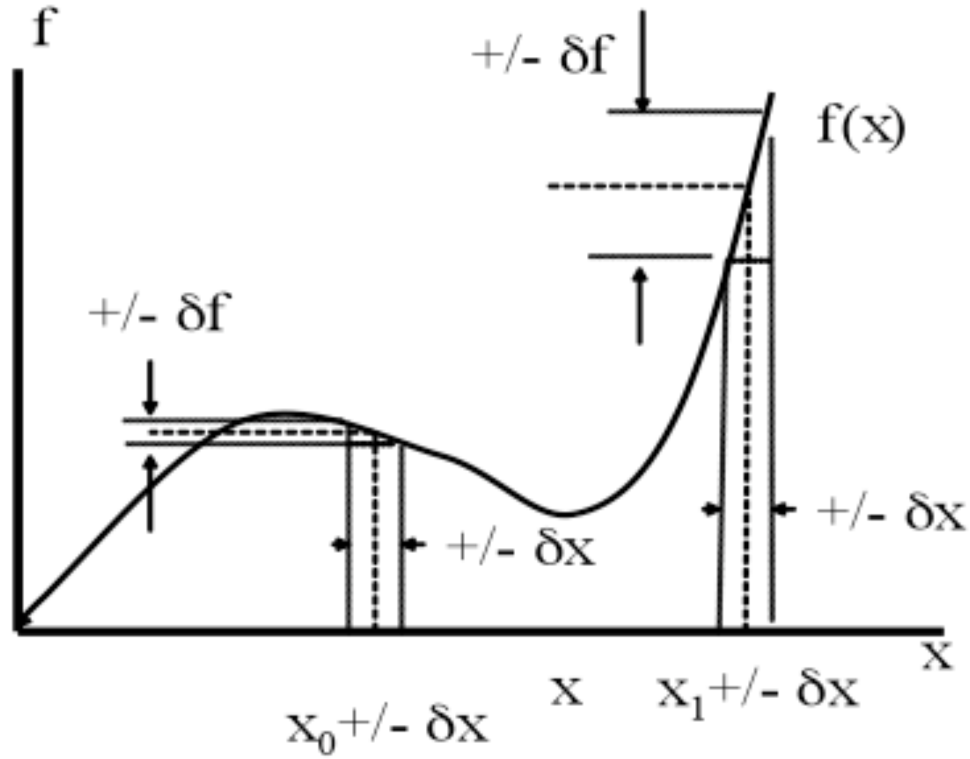
Graphically

x +/- δx

δf=?



The general rule: $\delta f = \left|\delta x \frac{df}{dx}\right|$

Another example

- Velocity V = X/T
  - X = 100 +/- 5 m   (5% fractional error; i.e. dX/X=0.05)
  - T = 10.0 +/- 0.5 s   (5% fractional error also)
  - dX and dT are uncorrelated

- dV/V is?

$$\delta V/V = \sqrt{\left(\delta X/X\right)^2 + \left(\delta T/T\right)^2}$$

$$= \sqrt{0.05^2 + 0.05^2} = \sqrt{2 \cdot 0.05^2} = \sqrt{2} \cdot 0.05 = 0.07$$

Exact numbers in formulas

- Exact numbers have zero error
- If C=BA
  - A=2.34 +/- 0.03 sec
  - B=2 (exact) +/- 0

$$\frac{\delta C}{C} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2}$$

$$\frac{\delta C}{C} = \sqrt{\left(\frac{\delta A}{A}\right)^2} = \frac{\delta A}{A} \qquad \delta C = C \bullet \frac{\delta A}{A} = 2A \bullet \frac{\delta A}{A} = 2\delta A$$

  - C=4.68 +/- 0.06 sec

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Answer

- Volume V=xyz
  - x= 20.00 +/- 0.02 cm
  - y= 5+/-1 cm
  - z = 8.0 +/- 0.4 cm $V = xyz = 20.00 \cdot 5 \cdot 8.0 = 800\text{cm}^3$

- Answer:

$$\delta V / V = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2} = \sqrt{\left(\frac{0.02}{20.00}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{0.4}{8.0}\right)^2}$$

$$= \sqrt{0.001^2 + .2^2 + 0.05} = 0.206 \approx 0.2$$

$$\delta V = 0.206 * 800 = 164.9 \approx 160\text{cm}^3$$

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Example: Simple Pendulum

- Period of pendulum T is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

  - where **L** is length of pendulum
  - **g** is the acceleration of gravity.

- Measure **L ± dL**, and **T ± dT**. What is d**g?**

- First, rewrite it: $$g = 4\pi^2 \frac{L}{T^2}$$

Turn the crank:

- Start:

$$\delta g = \sqrt{\left(\frac{\partial g}{\partial L}\delta L\right)^2 + \left(\frac{\partial g}{\partial T}\delta T\right)^2} \quad ; \qquad g = 4\pi^2 \frac{L}{T^2}$$

- So that:

$$\frac{\partial g}{\partial L} = 4\pi^2 / T^2 \quad ; \quad \frac{\partial g}{\partial T} = -2(4\pi^2 L / T^3)$$

$$\delta g = \sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3}\delta T\right)^2}$$

- And the *fractional* error is:

$$\frac{\delta g}{g} = \frac{1}{g}\sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3}\delta T\right)^2} \quad ; \quad \frac{1}{g} = T^2 / (4\pi^2 L)$$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{2\delta T}{T}\right)^2}$$

- **As given in the lab manual!**

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Weird functions:

- Don't panic– just apply the general rule!
  - For instance, $f(x,y)=\sin(xe^{ay})$
  - !!!!!!!!!!!!!!!

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + ...}$$

$$\frac{\partial f}{\partial x} = \cos(xe^{ay})e^{ay} \ , \quad \frac{\partial f}{\partial y} = \cos(xe^{ay})axe^{ay}$$

plug in your partial derivatives .....

$$\partial \mathrm{f} = \sqrt{(\cos(xe^{ay}\delta x)e^{ay})^2 + (\cos(xe^{ay})axe^{ay}\delta y)^2}$$

..... and you're done!

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