
Physics 1140 Fall 2012

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Lecture #2:

1. Significant figures (cont.)
2. Scientific Notation
3. Error notation
4. Simple error propagation in calculations

Advice for efficiency in the lab

1. **Read** the lab manual **before** doing the lab.
2. If uncertain about anything, **engage** the TA.
3. Goal for measurement: be as **accurate** and **precise** as possible within the limits of the hardware at hand.
4. Recipe for success:





“If you take the time it takes, the less time it will take.”

Or: *“Slow is smooth and smooth is fast.”*

Procedure for labs following M1 (pendulum):

1. Finish your M1 report within **7 working days**.
2. When finished: use **Reservation Book** to sign up for new experiment.
3. Choose ≥ 1 M#, ≥ 1 E#, and ≥ 1 O#, and one more.

Accuracy vs. Precision

	Accurate	Inaccurate (systematic error)
Precise		
Imprecise (reproducibility error)		

Known + Estimated Digits

- In general, when making a measurement, we record all the digits that we are certain of and then estimate ***one more digit.***
 - Known digits 2 and 7 are 100% certain
 - The third digit 6 is estimated (uncertain)
- In the reported length, all three digits (2.76 cm) are **significant** including the estimated one

Zero as a measured number



What is the length of the line?

First digit 4.?? cm

Second digit 4.5? cm

Last (estimated) digit is 4.50 cm

Significant Figures in Measurement

- Significant figures in a measurement include the known digits plus **one** estimated digit.

Counting Significant Figures

Number of Significant Figures

38.15 cm

4

5.6 ft

2

65.6 lb

122.55 m

Leading Zeros

Number of Significant Figures

0.008 mm

1

0.0156 oz

3

0.0042 lb

0.000202 mL

Trailing Zeros

Number of Significant Figures

25,000 in. 2

200 yr 1

48,600 gal 3

25,005,000 g _____

Trailing zeros in numbers without
decimals are **not significant**.

Scientific Notation

- “Scientific Notation” expresses a number in combination with a power of 10, so that the decimal is behind the **first nonzero digit**.
- e.g. $427.1 \times 10^4 = 4.271 \times 10^6$
 $0.0051 = 5.1 \times 10^{-3}$
 $1302 \times 10^{-5} = 1.302 \times 10^{-2}$
- Expressing a number in scientific notation should **not** change the number of significant figures.

Do NOT use scientific notation indiscriminately!

- Scientific notation is needed ONLY....
 - 1) ...if the number has ambiguous zeroes.
e.g. $150 \times 2.0 = 300 = 3.0 \times 10^2$
 - 2) ...if the number is (very) small (smaller than 0.01)
e.g. $0.003 = 3 \times 10^{-3}$
 - 3) ...if the number is already in exponential notation (but not scientific notation)
e.g. $24.831 \times 10^{56} = 2.4831 \times 10^{57}$

Errors Arise From A Number Of Sources Including:

- **Imprecision** - inherent error due to equipment or procedure (instrumental or method errors):
 - changing volume due to thermal expansion or contraction (temperature changes).
 - improperly calibrated equipment.
 - procedural design allows variable measurements.
- **Mistakes** – that you have made (to be avoided).
 - incomplete procedures.
 - reading scales incorrectly.
 - using the measuring device incorrectly.

How do we come up with an error estimate?

- *Judgment:*
 - Parallax in meter stick
 - Digits in voltmeter
 - Read the manual
- *Statistics:*
 - Scatter in repeated measurements
 - counts of random processes
 - We'll discuss in two weeks.
- *Propagate:*
 - Your estimated error through an equation

Reducing Error:

- Errors can often be detected by making **repeated** measurements.
- Error can be reduced by **testing** and **calibrating** equipment.
- The **average** or mean reduces data variations: it helps find a central value.

Significant Figures in Calculations

- A calculated answer cannot be more precise than the measuring tool.
- A calculated answer must match the least precise measurement.

Adding and Subtracting

- The answer has the same number of decimal places as the measurement with the **fewest** decimal places.

25.2 one decimal place
+ 1.34 two decimal places
Calculator gives 26.54

Answer: 26.5 (*one* decimal place)

- Round the calculated answer until you have the same number of significant figures as the measurement with the fewest significant figures.

Next: Quantitative Uncertainties

- Significant figures provide a **rough estimate** of the *uncertainty* in a value
- Better is to explicitly specify an estimated uncertainty:

– $m \pm dm = 4.06 \pm 0.02 \text{ kg}$

– $x \pm dx = 0.382 \pm 0.003 \text{ m}$

The “standard format” for uncertainties

- To write $x \pm \delta x$ in standard format
 - first round off δx to one (or two) significant figure(s)
 - then round off x to the leading power in δx .
 - In final result, value and error with the same power-of-ten-multiplier.
- Exception: If the leading digit in the error is "1", or at most "2":
 - In this case, you can keep two significant figures in the error.
 - Round the value to the same place as the second digit in the error.

Examples

- 62.6294 ± 0.0236 seconds
 - You rarely really know the *uncertainty* in this value so *precisely*.
 - Make a reasonable estimate of the precision with which you know the error;
 - E.g.: 62.63 ± 0.02 sec

- 3.1735 ± 0.0147 m/s
 - Instead (using the “1” rule), 3.174 ± 0.015 m/s

Propagating error estimates in calculations

- Addition and subtraction
 - Step 1: Do the math
 - Step 2: **new error is square root of the sum of the squares** of the individual errors

- $A+B$ with $A \pm dA$; $B \pm dB$:

- result is

$$(A + B) \pm \sqrt{\delta A^2 + \delta B^2}$$

- $A-B$:

- Result is

$$(A - B) \pm \sqrt{\delta A^2 + \delta B^2}$$

Propagating error estimates in calculations

- Example

- Measure A= 23.6 ± 0.2 cm ; B= 262.5 ± 0.5 cm

- Sum: (A+B) = 286.1 cm

- Error

- First, if the error in B is ± 0.5 cm, the total error *can't* be less

- But simply adding $0.5+0.2=0.7$ is generally too pessimistic (why?)

- Calculate:

$$\sqrt{0.5^2 + 0.2^2} = \sqrt{0.25 + 0.04} = \sqrt{0.29} = 0.58 \approx 0.6$$

- Important:

- This rule applies only for *uncorrelated* (random) errors

Master rule of error propagation:

- Given *uncorrelated* uncertainties dx , dy , dz ,.....
And a function $f(x,y,z....)$ the uncertainty in f is:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x} \delta x\right)^2 + \left(\frac{\partial f}{\partial y} \delta y\right)^2 + \left(\frac{\partial f}{\partial z} \delta z\right)^2 + \dots}$$

This might well be the single most important equation of this class.

Example: Simple Pendulum (first order approx)

- Period of pendulum T is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

- where L is length of pendulum
 - g is the acceleration of gravity.
- Measure $L \pm dL$, and $T \pm dT$. What is dg ?

- First, rewrite it:
$$g = 4\pi^2 \frac{L}{T^2}$$

Turn the crank:

- Start:

$$\delta g = \sqrt{\left(\frac{\partial g}{\partial L} \delta L\right)^2 + \left(\frac{\partial g}{\partial T} \delta T\right)^2} ; \quad g = 4\pi^2 \frac{L}{T^2}$$

- So that:

$$\frac{\partial g}{\partial L} = 4\pi^2 / T^2 \quad ; \quad \frac{\partial g}{\partial T} = -2(4\pi^2 L / T^3)$$

$$\delta g = \sqrt{\left(\frac{4\pi^2}{T^2} \delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3} \delta T\right)^2}$$

- And the *fractional* error is:

$$\frac{\delta g}{g} = \frac{1}{g} \sqrt{\left(\frac{4\pi^2}{T^2} \delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3} \delta T\right)^2} ; \quad \frac{1}{g} = T^2 / (4\pi^2 L)$$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{2\delta T}{T}\right)^2}$$

- **As given in the lab manual!**