Physics 1140 Fall 2011

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Lecture #2:

1. Significant figures (cont.)
2. Scientific Notation (Taylor ch 2)
3. Error notation (ch 2)
4. Simple error propagation in calculations (ch 3)

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Advice for efficiency in the lab

1. **Read** the lab manual **before** doing the lab.
2. If uncertain about anything, **engage** the TA.
3. Goal for measurement: be as **accurate** and **precise** as possible within the limits of the hardware at hand.
4. Recipe for success:

"If you take the time it takes, the less time it will take."

Or: *"Slow is smooth and smooth is fast."*

Procedure for labs following M1 (pendulum):

1. Finish your M1 report within **7 working days**.
2. When finished: use **Reservation Book** to sign up for new experiment.
3. Choose ≥1 M#, ≥1 E#, and ≥1O#, and one more.

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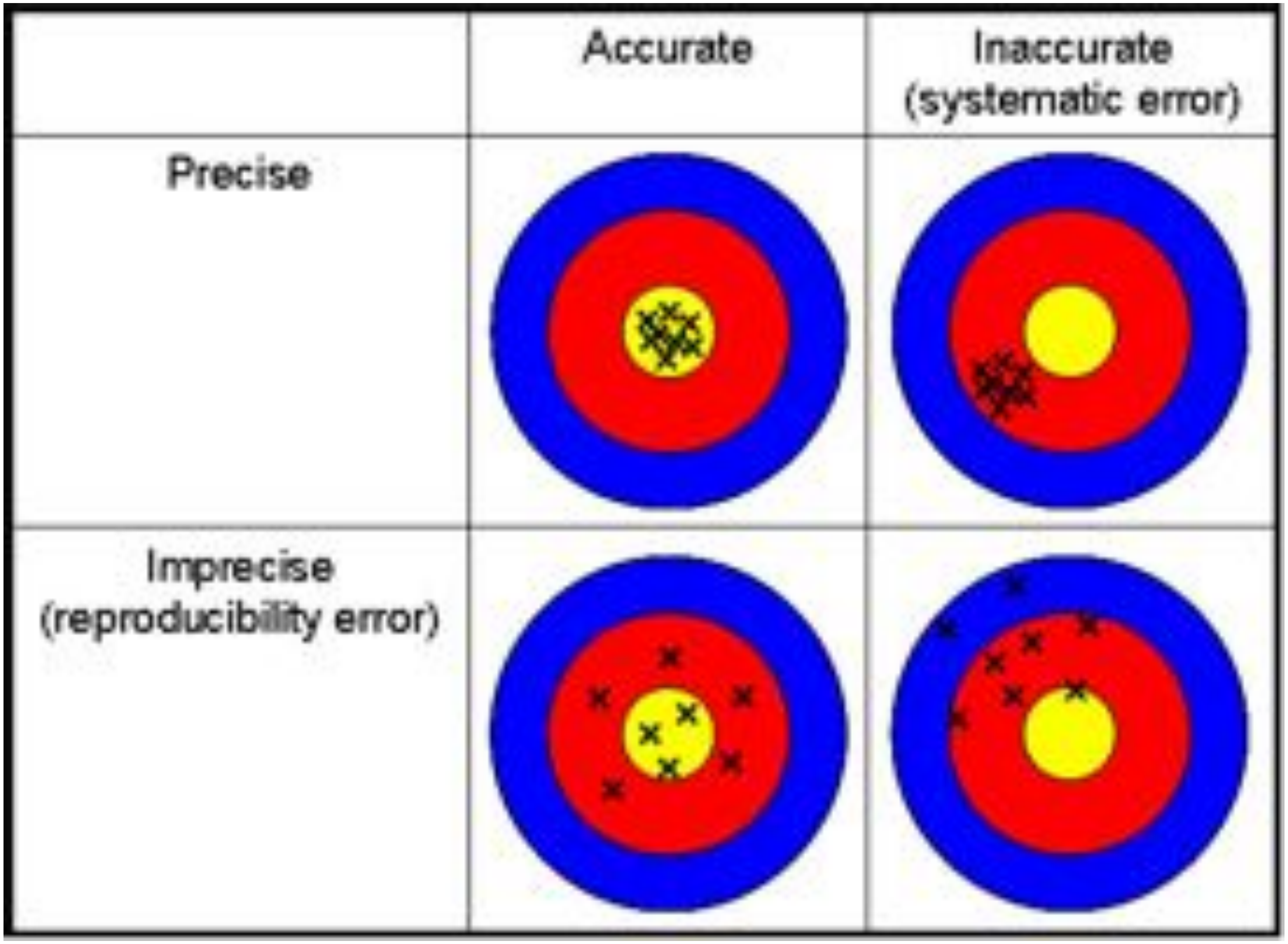
Accuracy vs. Precision



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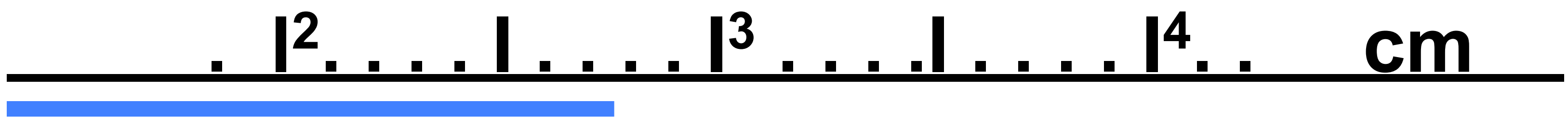
Simple measurement: Reading a Meter stick



First digit (known) = 2: 2.?? cm

Second digit (known) = 0.7 2.7? cm

Third digit (estimated) between 0.05- 0.07

Length reported = 2.75 cm

or 2.76 cm

or 2.77 cm

or (ideally) 2.76±0.01 cm

Known + Estimated Digits

- In general, when making a measurement, we record all the digits that we are certain of and then estimate ***one more digit***.
  - Known digits 2 and 7 are 100% certain
  - The third digit 6 is estimated (uncertain)
- In the reported length, all three digits (2.76 cm) are **significant** including the estimated one

Zero as a measured number

What is the length of the line?

First digit	**4.?? cm**
Second digit	**4.5? cm**
Last (estimated) digit is	**4.50 cm**

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Significant Figures in Measurement

- Significant figures in a measurement include the known digits plus one estimated digit.

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Counting Significant Figures

Number of Significant Figures

38.15 cm	4
5.6 ft	2
65.6 lb	___
122.55 m	___

Counting Significant Figures

Number of Significant Figures

38.15 cm	4
5.6 ft	2
65.6 lb	3
122.55 m	___

Counting Significant Figures

	Number of Significant Figures
38.15 cm	4
5.6 ft	2
65.6 lb	3
122.55 m	5

All non-zero digits in a measured number are **significant**.

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Leading Zeros

	Number of Significant Figures
0.008 mm	1
0.0156 oz	3
0.0042 lb	____
0.000202 mL	____

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Leading Zeros

	Number of Significant Figures
0.008 mm	1
0.0156 oz	3
0.0042 lb	2
0.000202 mL	____

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Leading Zeros

	Number of Significant Figures
0.008 mm	1
0.0156 oz	3
0.0042 lb	2
0.000202 mL	3

Leading zeros in decimal numbers are **not significant**.

Trailing Zeros

	Number of Significant Figures
25,000 in.	2
200 yr	1
48,600 gal	3
25,005,000 g	____

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Trailing Zeros

	Number of Significant Figures
25,000 in.	2
200 yr	1
48,600 gal	3
25,005,000 g	<u>5</u>

Trailing zeros in numbers without decimals are **not significant**.

Scientific Notation

- "Scientific Notation" expresses a number in combination with a power of 10, so that the decimal is behind the **first nonzero digit**.

- e.g. 427.1×10^4 = **4.271×10^6**
  0.0051 = **5.1×10^{-3}**
  1302×10^{-5} = **1.302×10^{-2}**

- Expressing a number in scientific notation should **not** change the number of significant figures.

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Do NOT use scientific notation indiscriminately!

- Scientific notation is needed ONLY....
  - 1) ...if the number has ambiguous zeroes.

    e.g. $150 \times 2.0 = 300 = 3.0 \times 10^2$

  - 2) ...if the number is (very) small (smaller than 0.01)

    e.g. $0.003 = 3 \times 10^{-3}$

  - 3) ...if the number is already in exponential notation (but not scientific notation)

    e.g. $24.831 \times 10^{56} = 2.4831 \times 10^{57}$

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Errors Arise From A Number Of Sources Including:

- **Imprecision** - inherent error due to equipment or procedure (instrumental or method errors):
  - changing volume due to thermal expansion or contraction (temperature changes).
  - improperly calibrated equipment.
  - procedural design allows variable measurements.

- **Mistakes** – that you have made (to be avoided).
  - incomplete procedures.
  - reading scales incorrectly.
  - using the measuring device incorrectly.

How do we come up with an error estimate?

- *Judgment*:
  - Parallax in meter stick
  - Digits in voltmeter
  - Read the manual

- *Statistics:*
  - Scatter in repeated measurements
  - counts of random processes
  - We'll discuss in two weeks.

- *Propagate:*
  - Your estimated error through an equation

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Reducing Error:

- Errors can often be detected by making **repeated** measurements.
- Error can be reduced by **testing** and **calibrating** equipment.
- The **average** or mean reduces data variations: it helps find a central value.

Significant Figures in Calculations

- A calculated answer cannot be more precise than the measuring tool.

- A calculated answer must match the least precise measurement.

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Adding and Subtracting

- The answer has the same number of decimal places as the measurement with the **fewest** decimal places.

25.2	one decimal place
+ 1.34	two decimal places

Calculator gives 26.54

Answer: 26.5 (*one* decimal place)

- Round the calculated answer until you have the same number of significant figures as the measurement with the fewest significant figures.

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Next: Quantitative Uncertainties

- Significant figures provide a **rough estimate** of the *uncertainty* in a value

- Better is to explicitly specify an estimated uncertainty:

  - m ± dm = 4.06 ± 0.02 kg

  - x ± dx = 0.382 ± 0.003 m

The "standard format" for uncertainties

- To write x ± δx in standard format
  - first round off δx to one (or two) significant figure(s)
  - then round off x to the leading power in δx.
  - In final result, value and error with the same power-of-ten-multiplier.

- Exception: If the leading digit in the error is "1", or at most "2":
  - In this case, you can keep two significant figures in the error.
  - Round the value to the same place as the second digit in the error.

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Examples

- 62.6294 ± 0.0236 seconds
  - You rarely really know the *uncertainty* in this value so *precisely*.
  - Make a reasonable estimate of the precision with which you know the error;
  - E.g.: 62.63 ± 0.02 sec

- 3.1735 ± 0.0147 m/s
  - Instead (using the "1" rule), 3.174 ± 0.015 m/s

Propagating error estimates in calculations

- Addition and subtraction
  - Step 1: Do the math
  - Step 2: **new error is square root of the sum of the squares** of the individual errors

- A+B with A±dA; B±dB:
  - result is

$$(A+B)\pm\sqrt{\delta A^2+\delta B^2}$$

  - A-B:
    - Result is

$$(A-B)\pm\sqrt{\delta A^2+\delta B^2}$$

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Propagating error estimates in calculations

- Example
  - Measure A= 23.6 ± 0.2 cm ; B= 262.5 ± 0.5 cm
  - Sum: (A+B) = 286.1 cm
  - Error
    - First, if the error in B is ± 0.5 cm, the total error *can't* be less
    - But simply adding 0.5+0.2=0.7 is generally too pessimistic (why?)
    - Calculate:

$$\sqrt{0.5^2 + 0.2^2} = \sqrt{0.25 + 0.04} = \sqrt{0.29} = 0.58 \approx 0.6$$

- Important:
  - This rule applies only for *uncorrelated* (random) errors

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Master rule of error propagation:

- Given *uncorrelated* uncertainties dx, dy, dz,…..
  And a function f(x,y,z….) the uncertainty in f is:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots..}$$

This might well be the single most important equation of this class.

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Example: Simple Pendulum (first order approx)

- Period of pendulum T is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

  - where **L** is length of pendulum
  - **g** is the acceleration of gravity.

- Measure **L ± dL**, and **T ± dT**. What is d**g?**

- First, rewrite it: $$g = 4\pi^2 \frac{L}{T^2}$$

Turn the crank:

- Start:

$$\delta g = \sqrt{\left(\frac{\partial g}{\partial L}\delta L\right)^2 + \left(\frac{\partial g}{\partial T}\delta T\right)^2} \quad ; \qquad g = 4\pi^2 \frac{L}{T^2}$$

- So that:

$$\frac{\partial g}{\partial L} = 4\pi^2 / T^2 \quad ; \quad \frac{\partial g}{\partial T} = -2(4\pi^2 L / T^3)$$

$$\delta g = \sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3}\delta T\right)^2}$$

- And the *fractional* error is:

$$\frac{\delta g}{g} = \frac{1}{g}\sqrt{\left(\frac{4\pi^2}{T^2}\delta L\right)^2 + \left(\frac{8\pi^2 L}{T^3}\delta T\right)^2} \quad ; \quad \frac{1}{g} = T^2 / (4\pi^2 L)$$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{2\delta T}{T}\right)^2}$$

- **As given in the lab manual!**

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