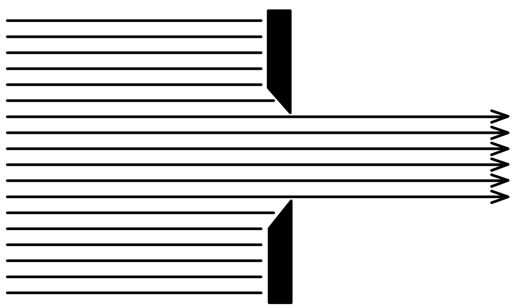
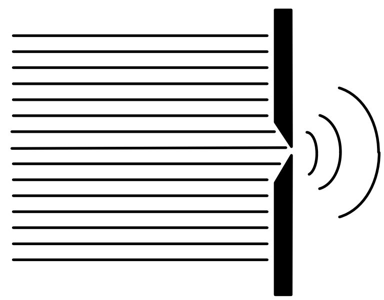
Lab O4: Single and Multiple Slit Diffraction

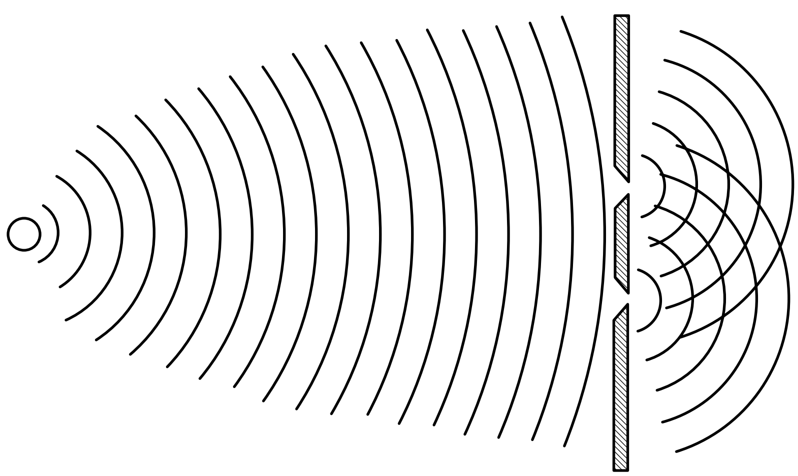
Light is a wave, an electromagnetic wave, and under the proper circumstances, it exhibits wave phenomena, such as constructive and destructive interference. The wavelength of light is too small to measure with ordinary tools, however, the *patterns made by interference* can be measured easily and from them you will find the wavelength of light.

The wavelength of light is about 600nm = 0.0006mm, and this wavelength λ sets the scale for the appearance of wave-like effects. For instance, if a broad beam of light partly passes through a wide slit, a slit which is very large compared to λ, then the wave effects are negligible, the light acts like a ray, and the slit casts a *geometrical shadow*. However, if the slit is small enough, then the wave properties of light become apparent and a diffraction pattern is seen projected onto a screen illuminated by the light from the narrow slit.

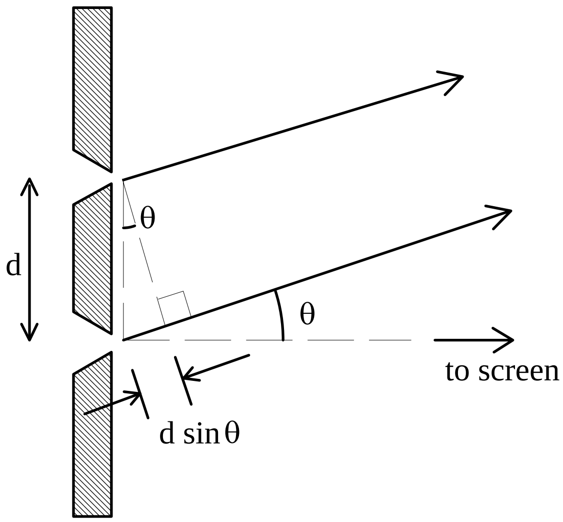




We now ponder the light from two *coherent* light sources a distance d apart. Coherent sources emit light waves that are *in phase*, or in sync. If we think of light like a water wave, we can imagine that the two sources each emit an identical succession of wave crests and troughs, with the both emitting crests at the same. One way to create such coherent sources is to illuminate a pair of narrow slits with a distant light source.



Consider the light rays from the two coherent point sources made from *infinitesimal* slits a distance d apart. We assume that the sources are emitting monochromatic light of wavelength λ. The rays are emitted in all forward directions, but let us concentrate on only the rays that are emitted in a direction θ toward a distant screen (θ measured from the normal to the screen, diagram below) . One of these rays has further to travel to reach the screen, and the *path difference* is given by $d\sin\theta$. If this path difference is exactly one wavelength λ or an integer number of wavelengths, then the two waves arrive at the screen in phase and there is constructive interference, resulting in a bright area on the screen. If the path difference is $\frac{1}{2}\lambda$, or $\frac{3}{2}\lambda$, etc., then there is destructive interference, resulting in a dark area on the screen.



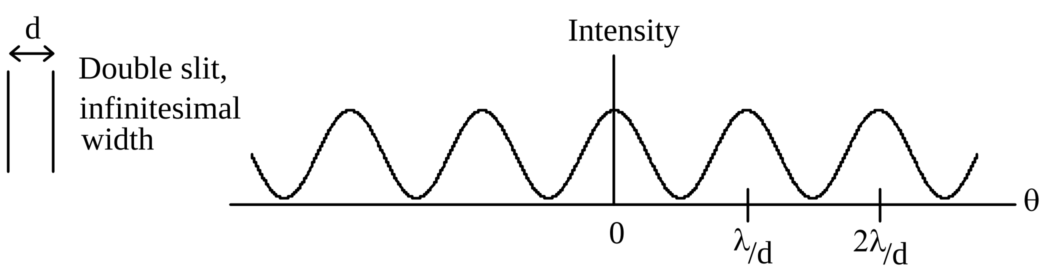
Bright: $d\sin\theta = n\lambda$

Dark: $d\sin\theta = (n+\frac{1}{2})\lambda.$

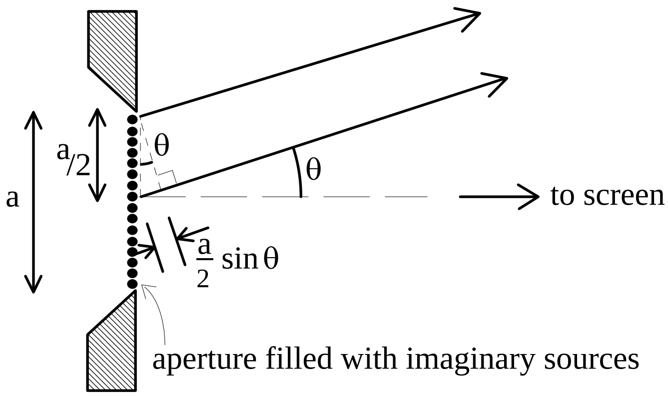
$\Big\}\; n = 0, \pm 1, \pm 2..$

If θ is small, then $\sin\theta \cong \theta$ (θ in rads!) and maxima occur on the screen at $\theta = n\frac{\lambda}{d}$; minima occur at $\theta = \left(n+\frac{1}{2}\right)\frac{\lambda}{d}$.

A complete analysis (not show here) yields a pattern of intensity vs. angle that looks like:



In fact, this regular-looking pattern is not observed in practice, because real slits always have finite width (not an infinitesimal width). We now ask what is the intensity pattern from a single slit of finite width a? *Huygens' Principle* states that the light coming from an aperture is the same as the light that would come from a collection of coherent point sources filling the space of the aperture. To see what pattern the entire array produces, consider first just two of these imaginary sources: one at the edge of the slit and one in the center. These two sources are separated by a distance a/2.

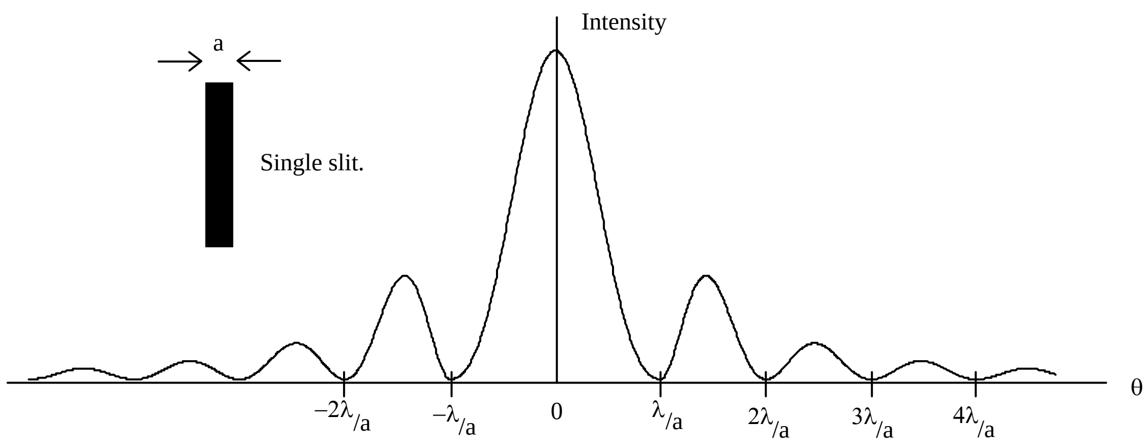


The path difference for the rays from these two sources, going to the screen at an angle θ, is $\frac{a}{2}\sin\theta$, and these rays will interfere destructively if $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$. But the same can be said for every pair of sources separated by a/2. Consequently, the rays from all the sources filling the aperture cancel in pairs, producing zero intensity on the screen when $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$

or, if θ is small,

$$\theta = \frac{\lambda}{a}. \qquad \text{(First minimum in single slit pattern.)}$$

A complete analysis (too complicated to show here) yields an intensity pattern, called a *diffraction* pattern, on the screen that looks like...

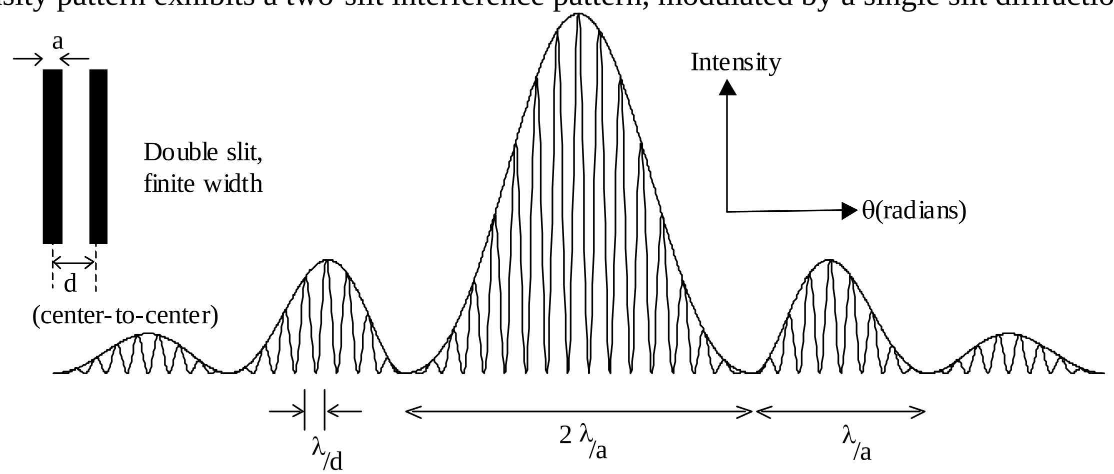


[The central maximum is actually much higher than shown here. It was reduced by a factor of 6, for clarity.] The single slit diffraction pattern has minima at

$$\theta = \pm\frac{\lambda}{a},\ \pm\frac{2\lambda}{a},\ \pm\frac{3\lambda}{a},\ \ldots \qquad \text{(Minima of single slit pattern.)}$$

So the separation of minima is λ/a, except for the first minima on either side of the central maximum, which are separated by 2λ/a.

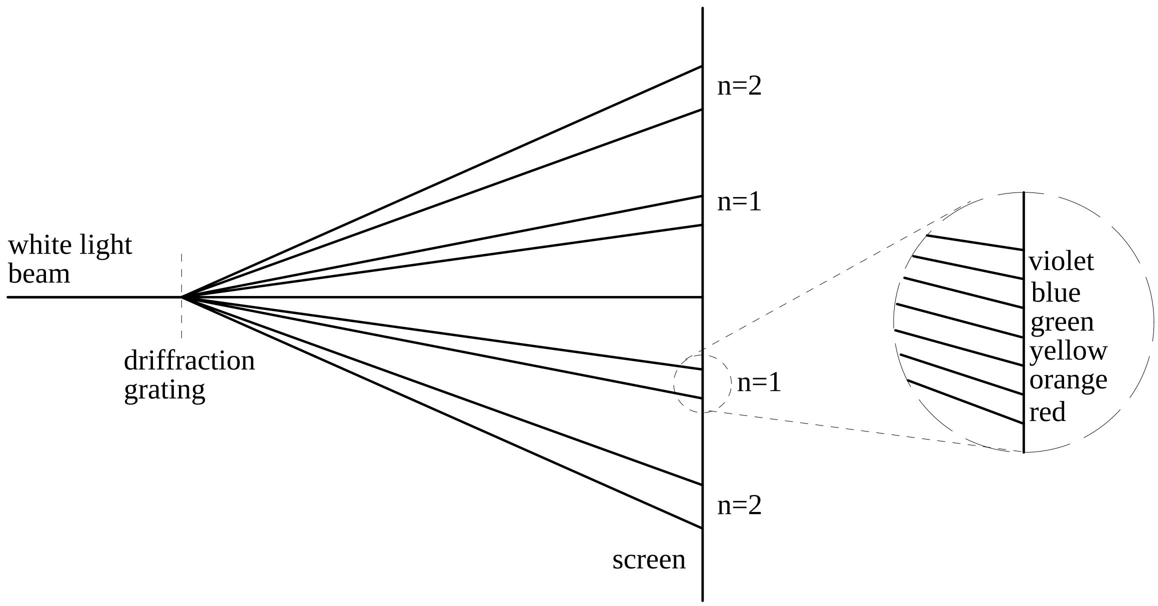
When the aperture consists of two finite slits, each of width a, separated by a distance d, the intensity pattern exhibits a two-slit interference pattern, modulated by a single slit diffraction pattern:



In this full pattern, the finely spaced interference maxima are spaced $\Delta\theta = \frac{\lambda}{d}$ apart, while the more widely spaced minima of the single-slit diffraction pattern are separated by $\Delta\theta = \frac{\lambda}{a}$ or $\frac{2\lambda}{a}$. Note that an interference maximum can be wiped out if it coincides with a diffraction minimum.

An aperture consisting of many slits of uniform width and spacing is called a *diffraction grating*. Using exactly the same arguments as above, one can show that a diffraction grating, whose slit separation is d, produces maxima with a separation of $\Delta\theta = \frac{\lambda}{d}$, the same as a double slit, except the maxima from a grating are very cleanly separated and sharply peaked. The more slits in the grating, the more sharply peaked the maxima.

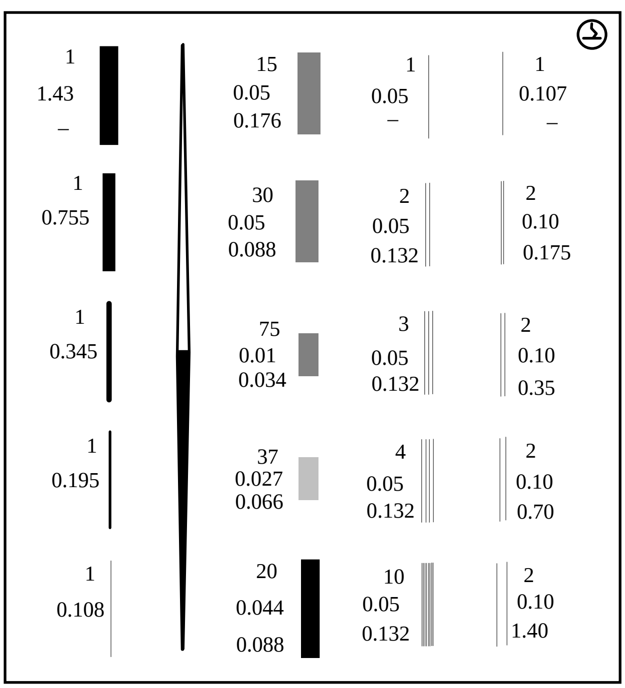
So far, we have discussed the case of monochromatic light, light with a single wavelength λ. White light, however, is a mixture of wavelengths from deep red ($\lambda_{red} \cong 700\text{nm}$) to violet ($\lambda_{violet} \cong 400\text{nm}$). For a diffraction grating or double slit, the angle of the n^{th} maximum from the central peak is proportional to λ: $\theta_n = n\frac{\lambda}{d}$. A white light source thus produces not just one maximum for each n, but a whole spectrum of colors for each n.



Experiment

The light source in parts 1 and 2 of this experiment is a He-Ne laser which produces a monochromatic beam with a wavelength of λ = 632.8 nm and a beam diameter of about 1 mm . The power output of our lasers is about 1 mW, a small amount, but still enough to damage your retina if you look directly into the beam.

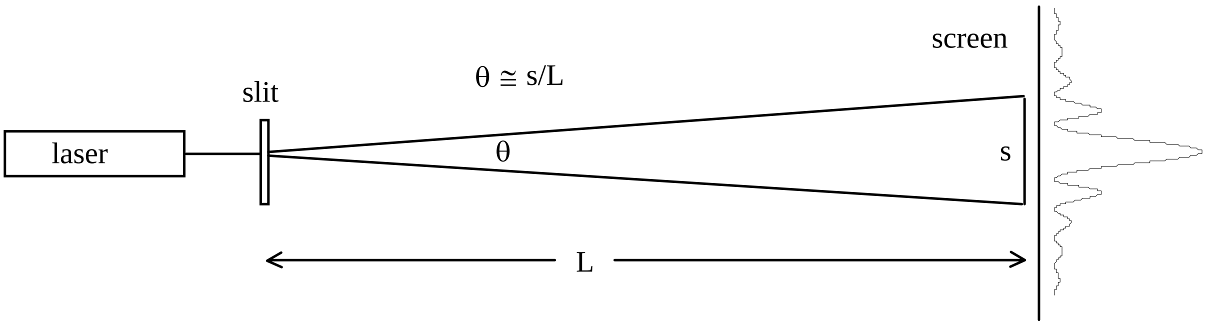
NEVER LOOK INTO A LASER BEAM.



The aperture consists of an opaque photographic negative, containing several single, double, and multiple slits. The arrangement of slits on the plate is shown here. The numbers are those given by the manufacturer and are ***not*** accurate.

Number of lines N.
Width a in mm.
Spacing d in mm.

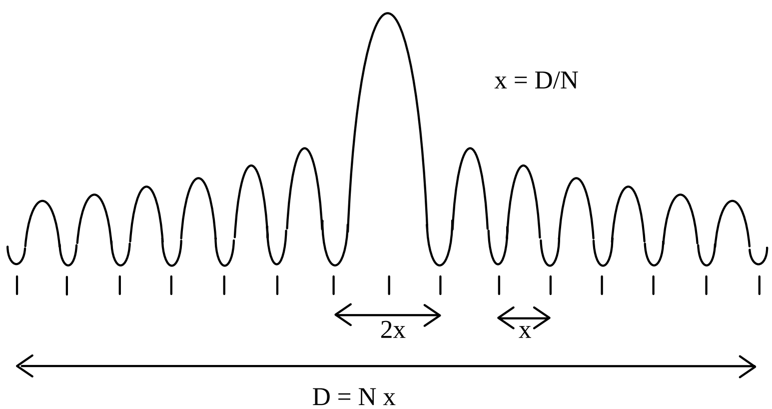
Tape a piece of paper on the wall to use as the screen. Then shine the laser through the various apertures and observe the diffraction patterns. Note the appearance of the single, double, and multiple slit patterns. In particular, observe how the single slit pattern changes as you vary the width of the slit.



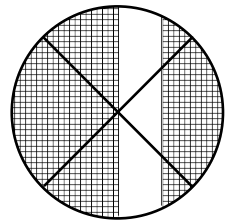
This diagram shows the relation between the width s of some feature (any feature) on the screen, the angular width θ of that feature, and the distance L from the aperture to the screen.

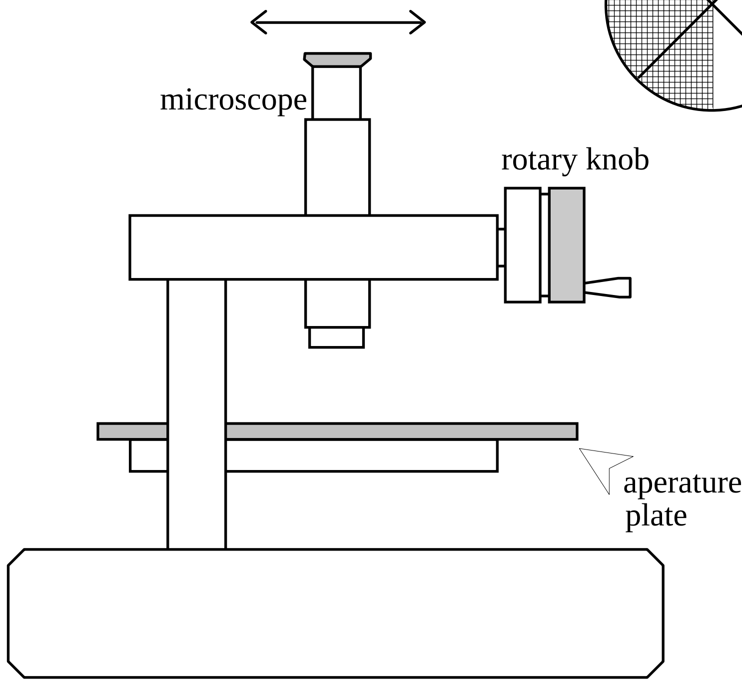
Part 1. Single Slits

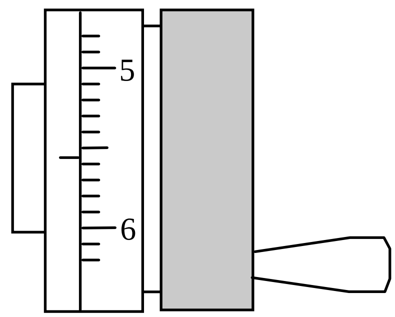
In this part we will test the relation $\theta = \frac{x}{L} = \frac{\lambda}{a}$, where x is the separation of minima in the single-slit diffraction pattern. Measure the distance L from the slit to the screen. Observe the diffraction pattern on your paper screen for each of the four smaller single slits(the four with approximate widths a = 0.10, 0.20, 0.35, and 0.75mm). With a pencil, mark the positions of as many of the minima that you can see and measure the spacing x between adjacent minima on the screen, for each of the four slits. To do this most accurately, measure the width of the entire pattern and divide by the number of maxima in the pattern (central max counts as two!).



Using the measuring microscope, measure the width a of each of the four slits. The measuring microscope is mounted on a platform which can be moved horizontally by turning a rotary knob. A scale on the rotary knob indicates the microscope's position. The smallest divisions on the scale are 0.01mm, but one can interpolate to a tenth of a division (0.001mm). Measure each slit twice: once moving the microscope from left to right and again from right to left. When measuring, one must always move in the same direction, to avoid gear backlash. If you overshoot the slit edge, back up and re-approach from the same direction.







Position = 0.556 mm

From your data, compute $\lambda = a\frac{x}{L}$ for each of the four slits. Also compute the mean and standard deviation of the mean, and compare your results with the known wavelength $\lambda_{\text{known}} = 632.8\,\text{nm}$.

$$\bar{\lambda} = \frac{1}{N}\sum_i \lambda_i \qquad \sigma = \sqrt{\frac{\sum_i \left(\lambda_i - \bar{\lambda}\right)^2}{N-1}} \qquad \sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}$$

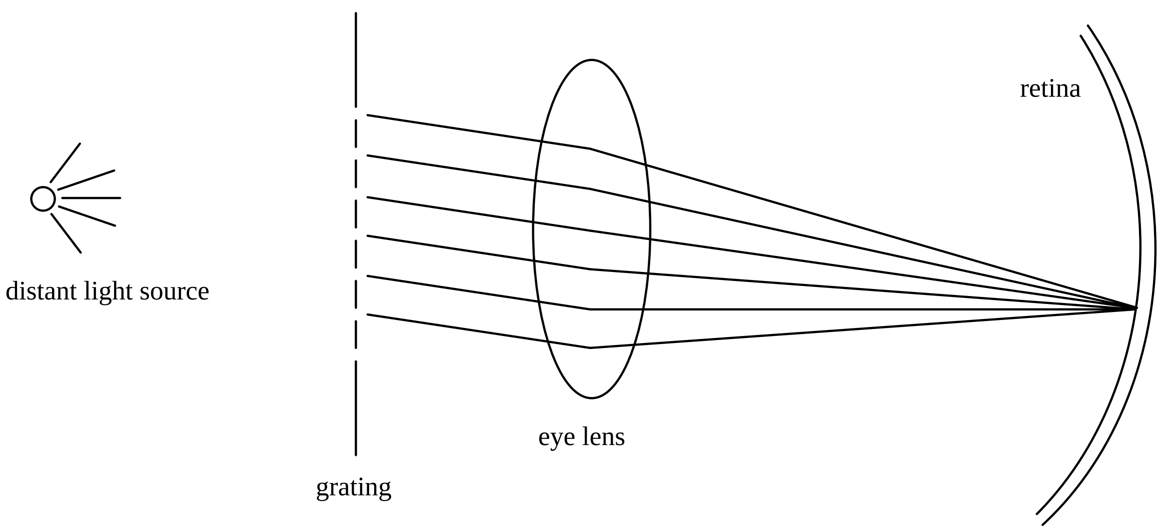
Finally, with your 4 points, make a plot of L/x vs. a/λ_{known} and on the same graph plot the line y = x. As usual, comment on any discrepancies between theory and experiment.

Part 2. Double Slit

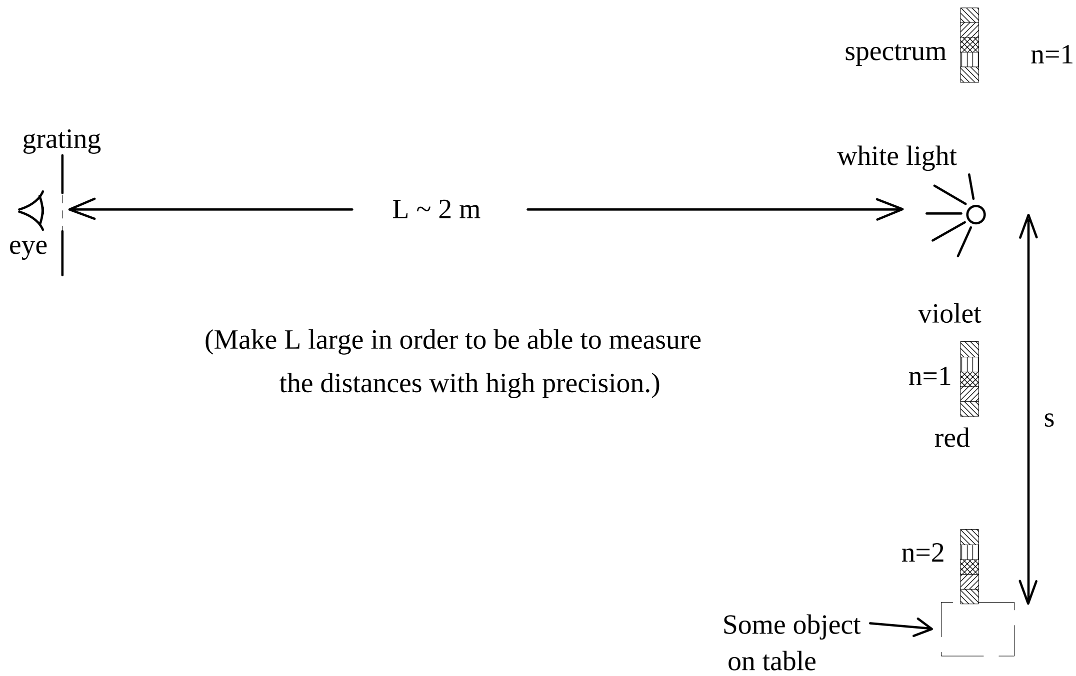
Repeat the above procedure with the one double slit aperture which has a nominal slit separation of 0.35 mm (the one in the middle). Assume that the wavelength λ=632.8nm is known, and make measurements of the diffraction pattern which allow you to compute d, the separation of the slits. *(d is the distance between the slit centers but it is easier to measure the distance from the left side of one slit to the left side of the other slit.)* Also measure d directly with the measuring microscope. Compare your two values for d.

Part 3. Diffraction Grating

For this part, use the 75 slit diffraction grating in the center of the aperture plate. The spacing of the slits is d = 0.034 mm. Turn on the incandescent lamp with the straight, vertical filament, and turn up the Variac autotransformer to make a bright white light source. Hold the grating close to your eye and look directly through the grating at the source. You will see the several spectra, corresponding to the various n's. Why is it that you see the spectra so clearly? It is because your eye lens focuses each parallel bundle of rays, corresponding to each particular color, onto your retina.



Notice that the red end of each spectrum is on the outside, away from the source, and the blue end is on the inside, toward the source. Notice also, that by moving toward and away from the source, you can make the spectra appear to move inward or outward with respect to objects on the table near the lamp. Choose one of the spectra, say n = 3 or 4, and position yourself so that the red edge of the spectrum appears to coincide with the edge of something on the table. Measure the distance s from that edge to the source and the distance of your eye from the source. From these distances compute the angle θ. From θ, n, and the known spacing of the grating, compute the wavelength λ of light at the red end of the range of human vision. Compare with the spectrum chart on the wall.



PreLab Questions:

1. (Counts as two questions) What is the angular width *in radians* of the central maximum of a single slit diffraction pattern (width = distance from 1st minimum to 1st minimum on either side of the central maximum)? What is the physical width x on the screen (not the angular width) of the central maximum? Give algebraic answers and define the symbols used. Show with a diagram the relation between the angular width θ of the central maximum, its physical width x on the screen, and the distance L from the slit to the screen.

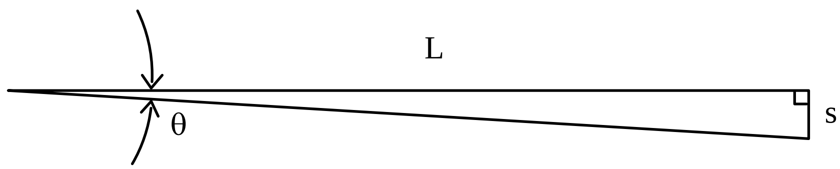
2. What is Huygens' Principle?

3. A single slit with a width a = 0.080 mm, is illuminated with a He-Ne laser. The diffraction pattern is projected onto a screen a distance L = 2.00 m away. What is the distance on the screen between the n = 4 minimum to one side of the central max and the n = 4 minimum on the other side.

4. A double slit with separation d = 0.25 mm and negligibly small widths (very small a) is illuminated with a He-Ne laser. The interference pattern is projected onto a screen a distance L = 1.50 m away. What is the distance (in mm) on the screen between the n = 2 maximum on one side of the central max and the n = 3 maximum on the other side? Make a qualitative sketch of the pattern, indicating the central maximum and the distance between the n=2 max on one side and the n=3 max on the other.

5. What is an aperture with many slits called?

6. How are s, L, and θ, in the diagram below, related? (That is, write an algebraic relation showing how s, L, and θ are related.) A student measures distances s = 20.0 cm and L=200.0 cm and then computes the angle θ. What percentage error is made by assuming $\theta \cong \frac{s}{L}$, rather than computing θ exactly. As usual, show your work.



7. As in the diagram on p.O4.9, a white light beam passes through a diffraction grating with a slit spacing of $d = 0.010$ mm and several rainbow spectra are projected onto a screen a distance $L = 1.500$ m away. How far from the center of the screen (the $\theta = 0$ position) is the red end of the n=2 spectrum? How far from the center is the violet end of the n=2 spectrum? The wavelength of red light is approximately 700 nm; the wavelength of violet light is 400 nm.

8. Make a qualitative sketch showing what the graph of L/x vs. a/λ should look like in part 1. What is the slope of this plot?

9. True or False: it is perfectly safe to stare into a laser.