

Lab O3: Snell's Law and the Index of Refraction

Introduction.

The bending of a light ray as it passes from air to water is determined by Snell's law. This law also applies to the bending of light by lenses and to the guiding of light by the fiber optic cables that carry modern communications signals. In part 1 of this lab you will investigate the bending of light as it travels from air to plastic and plastic to air. Your data will illustrate Snell's law. In part 2, you will find that the speed of light in plastic varies slightly with the wavelength.

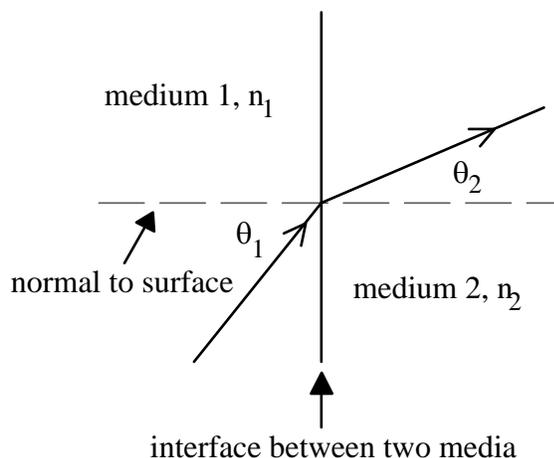
The speed of light in a vacuum is $c = 3.00 \times 10^8$ m/s. Because of a complex interaction between an electromagnetic wave (light) and the charges in matter (electrons and protons), a light signal travels more slowly in transparent materials, such as glass or water, than in vacuum. The ratio of c , the speed of light in a vacuum, to v , the speed of light in a medium, is called the index of refraction, n , of the medium

$$(1) \quad n = \frac{c}{v}.$$

medium	n
vacuum	1
air	1.000293
water	1.333
CF ₄	1.461
diamond	2.419

Note that $n \geq 1$, always. In most applications, the index of refraction of air can be taken to be one, with negligible error.

A ray of light passing from one medium to another (say, from air to water) is bent or refracted from its original path according to Snell's Law, which states



$$(2) \quad n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

where n_1 and n_2 are the indices of refraction in medium 1 and medium 2, and the angles are measured with respect to the normal (the perpendicular) to the interface. The light is always bent more toward the normal in the material with the larger n , and, in the diagram to the left, we have drawn the angles for the case $n_2 > n_1$. Note that if the ray strikes the interface perpendicularly, at an angle of $\theta_1 = 0^\circ$, then it is not refracted and exits perpendicularly with $\theta_2 = 0^\circ$.

Although Snell's Law can be proven from fundamental principles, we will consider it to be an experimental fact and use it to determine the index of refraction of Lucite (a transparent plastic).

Consider a ray of light passing from air ($n_{\text{air}} = 1$) into another medium such as Lucite with an index of refraction n ($n > 1$). In this case, the ray is incoming (incident) on the air side and outgoing (refracted) on the Lucite side of the interface, and we have

$$(3) \quad \sin \theta_i = n \sin \theta_r, \quad n = \frac{\sin \theta_i}{\sin \theta_r} \quad (\text{air} \rightarrow \text{Lucite})$$

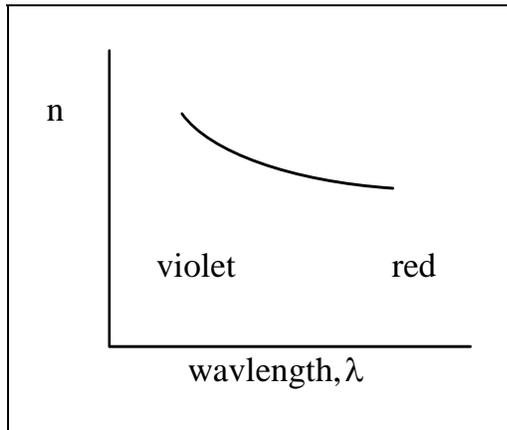
where θ_i and θ_r are the angles of the incident and refracted rays, respectively.

Suppose now a ray of light passes from a medium (Lucite) with index of refraction $n > 1$ into air. We then have

$$(4) \quad n \sin \theta_i = \sin \theta_r, \quad n = \frac{\sin \theta_r}{\sin \theta_i} \quad (\text{Lucite} \rightarrow \text{air})$$

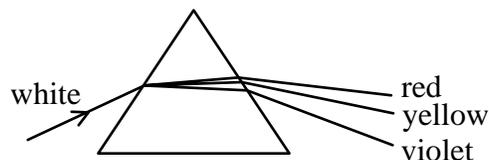
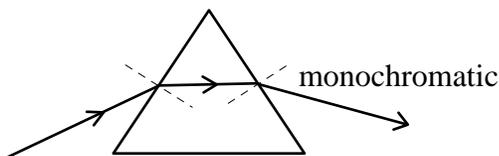
In the two cases just considered, if instead of labeling the angles as θ_i and θ_r , we label the angles θ_a (θ in air) and θ_p (θ in Lucite plastic), then both formulas (3) and (4) become

$$(5) \quad n = \frac{\sin \theta_a}{\sin \theta_p}$$



The index of refraction is, in general, a function of the wavelength of the light. It turns out that n is smaller for long wavelength (red) light and larger for short wavelength (blue) light. That is, blue light is bent more by glass or plastic lenses and red light is bent less. This is the cause of chromatic aberration in simple lenses. Multi-element lenses which correct for chromatic aberration are called achromats and are rather expensive.

This spread in the values of n , or *dispersion*, is the cause of the rainbow of colors produced by a prism. This effect was first studied quantitatively by Isaac Newton when he was still a student.

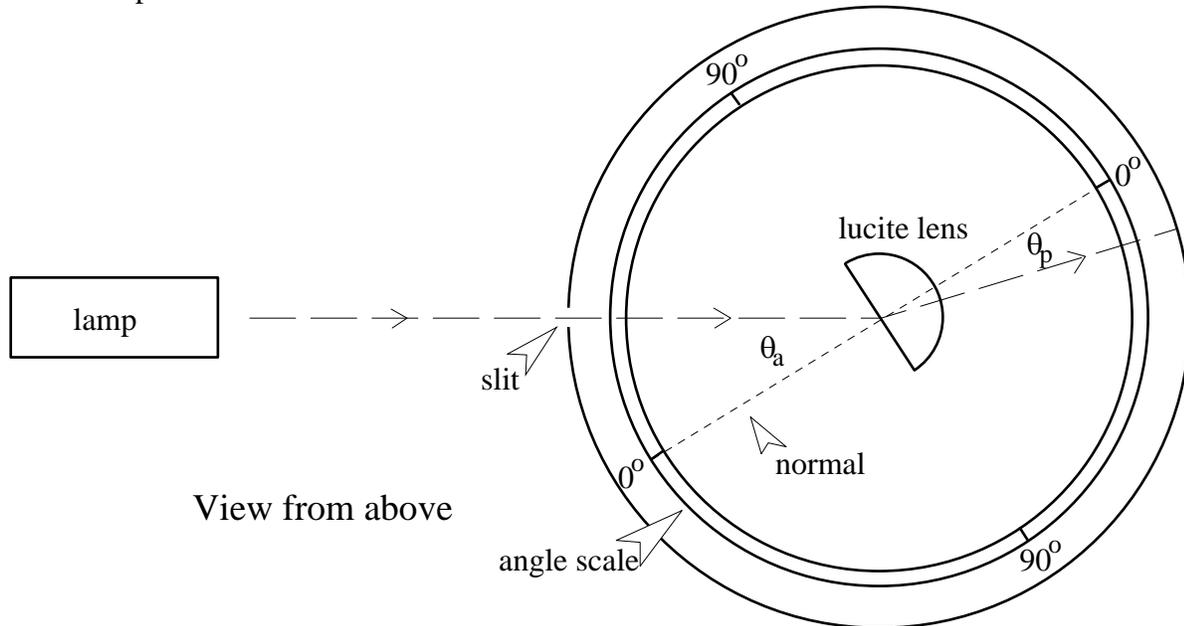


The colors of the spectrum, in order from longest to shortest wavelength are: red, orange, yellow, green, blue, violet.

Procedure

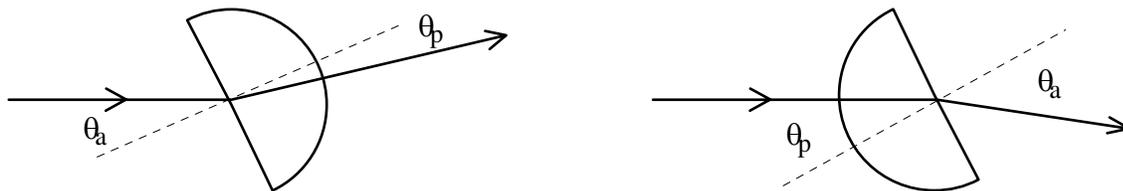
In this lab, using a semicircular Lucite lens, we will measure the angles θ_a and θ_p as a ray of light passes between the flat side of the Lucite and air. The angles are easily measured because the lens sits

at the center of a circular platform with an angle scale along its perimeter. A ray of light, defined by a slit on the side of the platform, passes through the lens and the angles θ_a and θ_p are read by noting where the shaft of light touches the angle scale. The apparatus is shown below. The lamp and the slit are fixed in position, while the lens and the platform with the angle scale can be rotated with a handle under the platform.



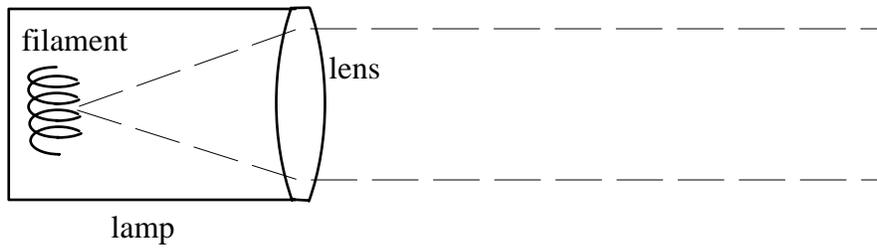
Note that refraction of the light ray takes place only on the flat side of the semi-circular lens. On the circular side, the ray strikes the interface exactly normal to the surface and there is no refraction.

Recall that θ_a is the angle on the *air* side of the flat interface and θ_p is the angle of the *plastic* side of the interface. So if the flat side is facing the slit, θ_a is the incident angle, but if the curved side is facing the slit, then θ_p is the incident angle.

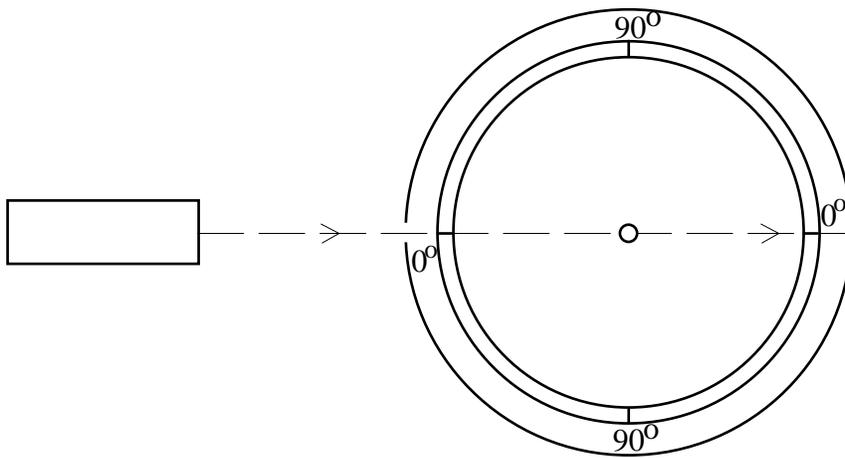


Before taking measurements, it is crucial that the lamp and the lens be carefully positioned with respect to the angle scale platform. Unless the alignment is perfect, your data will have systematic errors. The alignment procedure is as follows:

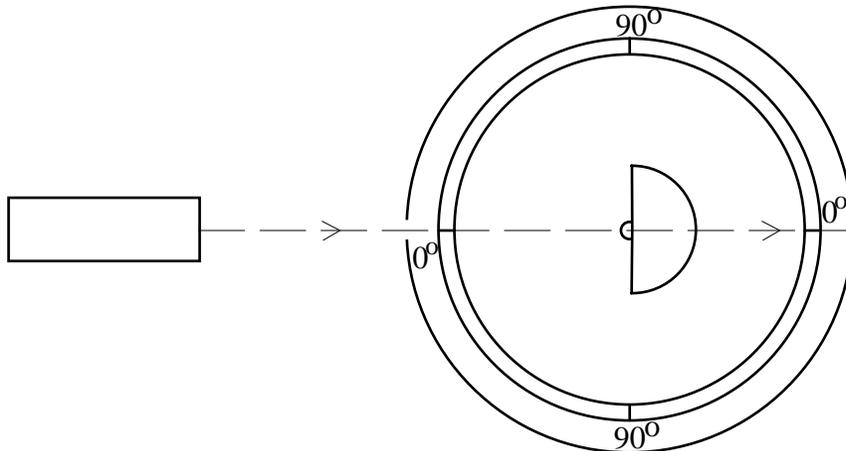
1. Begin by adjusting the lamp to give a good parallel beam. Point the lamp at a distant wall (at least 30 feet away) and observe the image of the light bulb filament. Slide the bulb back and forth slightly, with the knob on the underside of the lamp, to get a sharp image of the filament on the distant wall. Adjust the orientation of the lamp to make the image of the filament vertical (so that maximum light passes through the vertical slit on the platform).



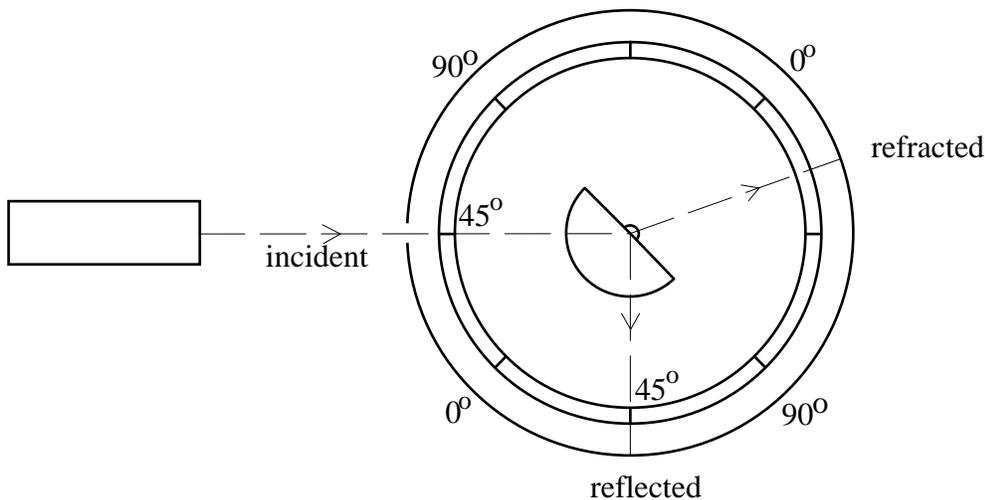
2. Remove the Lucite lens from the platform and then position the lamp and rotate the platform so that a fine, sharp shaft of light is seen entering at 0° , passing over the exact center of the platform, and exiting at 0° . You have to get the light slightly higher than the platform and angle it down slightly to get a bright shaft of light showing on the platform.



3. Now place the Lucite lens on the platform with the flat edge centered on the platform and perpendicular to the 0° line. Adjust the position of the lens slightly until the ray of light exits at 0° when it enters at 0° . Check that this occurs when both the flat side is toward the slit and when the platform is rotated 180° so that the rounded side is toward the slit.

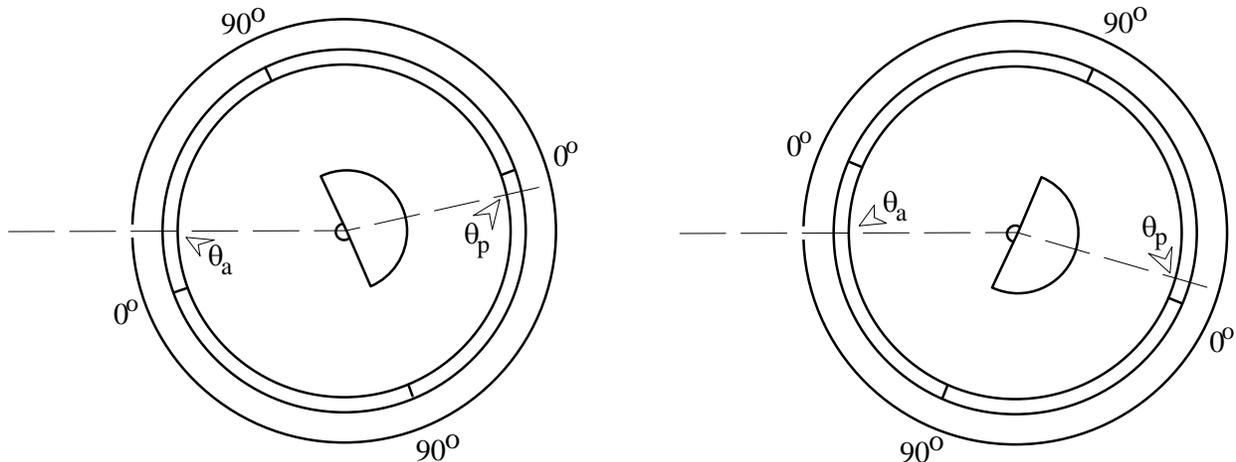


4. As a further check of the alignment, rotate the platform to the 45° position and observe the reflected ray, which should also be at 45° .



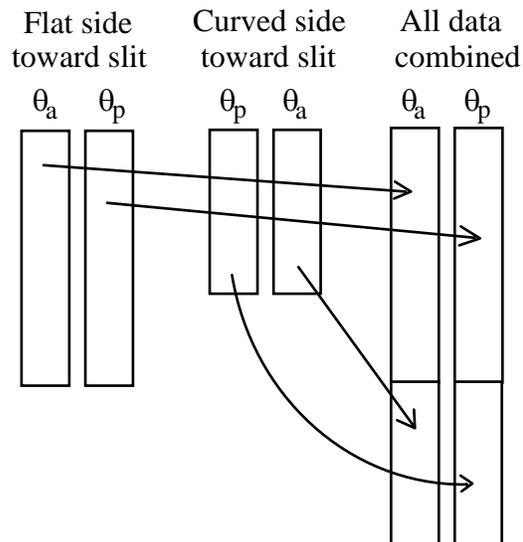
Part 1. Index of Refraction of Lucite

At last, you are ready to take data. With the flat side toward the slit, take several measurements of θ_a and θ_p for various θ_a 's from 5° up to about 70° . Estimate the angles to the nearest 0.1° . For each value of θ_a , take two measurements of θ_p for both possible orientations of the lens, as shown below, and average your two values for θ_p . If your alignment is good, the two values should be very close, within a few tenths of a degree. Record both θ_p 's in your handwritten notes, but enter only the average θ_p into your Mathcad document.



Now repeat all these measurements, with the circular side of the lens toward the slit. Take data for values of θ_p from 5° to about 40° , every 5° or so. When θ_a is near 75° , which occurs when θ_p is near 40° , you will notice that the refracted beam is no longer a narrow beam of white light, but has broadened into a rainbow of colors. Use a white piece of paper as a screen to see this clearly. Use the position of the middle of the broadened light beam to determine θ_a .

Now combine all your data to make a single table of θ_a and θ_p .



To make the new arrays with the combined data, simply enter the data again. You can avoid much tedious typing by using copy and paste. Be sure that all the air side angles are in one column and all the plastic side angles are in the other column.

Using all the data make a plot of θ_a vs. θ_p , with both angles in degrees. Now *convert the angles from degrees to radians*, and then make a plot of $\sin(\theta_a)$ vs. $\sin(\theta_p)$. [The default mode of Mathcad assumes that angles are in radians when computing sine and cosine.] The plot should be a straight line.

For each θ_a / θ_p pair, compute $n = \frac{\sin(\theta_a)}{\sin(\theta_p)}$, and graph n vs. θ_a . The index of refraction n

should be a constant, independent of θ_a ; however, for small θ , the fractional errors are large, and you will probably observe large deviations of n from the average value n_{avg} when θ is small. [The curve of n vs. θ_a should be flat, except for random fluctuations; if not, there is some systematic error, probably due to misalignment.]

By examining the graph of n vs. θ_a , decide which small- θ values of n to throw out, and then use your “good” data to compute the mean of n , standard deviation, the standard deviation of the mean.

$$n_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N n_i \quad \sigma = \sqrt{\frac{\sum_{i=1}^N (n_i - n_{\text{avg}})^2}{N-1}} \quad \sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}$$

Part 2. Dispersion of Lucite

In this part, you will measure n_{red} and n_{violet} , the values of n at the *extreme* red and violet ends of the visible spectrum. To do this, orient the platform so the circular side of the lens is facing the slit, and then adjust the angle so that the refracted beam is close enough to 90° to give a good spectrum of colors, but not so close that some of the colors are lost to internal reflection. Make measurements of the angles of the refracted beam at the red and violet edges of the spectrum, θ_{red} and θ_{violet} , as well as

$\theta_{\text{incident}} = \theta_p$. Compute

$$n_{\text{red}} = \frac{\sin(\theta_{\text{red}})}{\sin(\theta_p)}, \quad n_{\text{violet}} = \frac{\sin(\theta_{\text{violet}})}{\sin(\theta_p)}.$$

Make measurements at 2 or 3 slightly different incident angles and repeat each measurement for the two orientations of the lens.

You now have three values of n for three different wavelengths: n_{avg} in the middle of the visible spectrum (yellow), n_{red} , and n_{violet} . Look at the spectrum chart on the wall to estimate the wavelengths of the red, yellow, and violet light. Make a plot of n vs. wavelength with your three points.

PreLab Questions:

1. What is Snell's Law? Your answer should include a diagram.
2. A ray of light in air strikes the surface of a pool of water at an angle of 30° from the normal. What is the angle (measured from the normal) of the refracted ray in the water?
3. In this experiment, why is there no refraction of the light ray on the circular side of the Lucite lens?
4. When a light ray passes from a material of higher n to a material of lower n (for instance, a light ray passing from glass to air), the refracted angle is larger than the incident angle. In this case, the angle of

the incident ray can exceed a *critical angle*, at which the angle of the refracted ray is 90° . For incident angles greater than the critical angle, there is no refracted ray, only a reflected ray. This phenomenon is called total internal reflection. What is the critical angle for water with $n = 1.33$? What is the critical angle for glass $n = 1.50$?

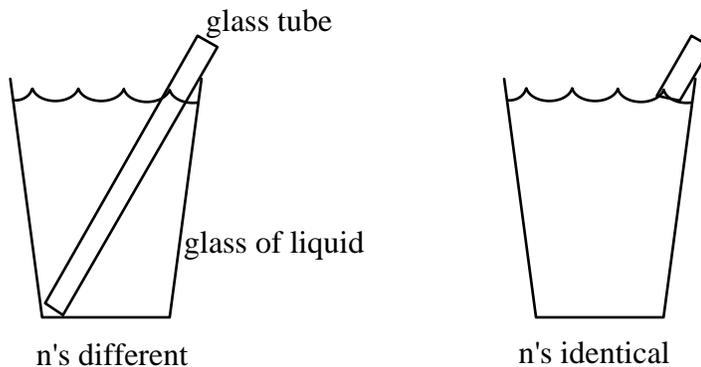
5. Show with a sketch what the graph of $\sin(\theta_a)$ vs. $\sin(\theta_p)$ for Lucite should look like (for a given fixed wavelength λ). Your sketch should show the shape of the curve (linear, parabolic, etc.) as well as indicate whether the curve goes through the origin. What can you say about the slope of this curve?

6. Suppose you measured n vs. θ_a using a red light source and a blue light source. On the same graph, sketch both n_{red} vs. θ_a and n_{blue} vs. θ_a , where n_{red} is the index measured with red light and n_{blue} is the index measured with blue light. (Qualitative sketch only! No numbers.)

7. What does a graph of n vs. wavelength look like for glass or plastic?

8. When the curved side of the lens is toward the slit, which angle is θ_a ? Which angle is θ_p ? Indicate your answers with a diagram.

9. If a piece of clear glass with an index of refraction n is placed in a clear liquid with the same index of refraction n , the glass becomes completely invisible. Why?



10. When a light ray passes from air through the Lucite semi-circular lens and back into air the ray path appears as shown below, with $\theta_a > \theta_p$. Suppose that a Lucite lens with $n = 1.55$ is submerged in a clear liquid with $n_{\text{liq}} = 1.70$, so the ray path is now liquid \rightarrow Lucite \rightarrow liquid, and the angle θ_a should now be labeled θ_{liq} . Is θ_p greater than, less than, or equal to θ_{liq} ? Suppose $\theta_{\text{liq}} = 20^\circ$, what is θ_p ?

