Lab O2. Variation of Atmospheric Pressure with Altitude

Introduction

In this lab you will investigate the ideal gas law. By moving a plunger in a sealed container of air, you will find the variation of pressure with volume. By warming the air with your hands, you will investigate the variation of pressure with temperature. Then, by riding up and down in the elevator with your apparatus, you will investigate the variation of pressure with altitude.

We all live near the bottom of an ocean of air. At sea level, the weight of the air overhead presses on us with a pressure of $\cong 10^5 \, \mathrm{N/m^2} = 14.7 \, \mathrm{lbs/in^2}$ (12.5 psi in Boulder). We are not aware of the this great weight because the air presses on us from all sides, even from our insides (due to the air in our lungs).

At higher altitudes, there is less air and less weight overhead, and the pressure is less. Also, because air is readily compressible, the lower layers of air are compressed by the weight of the air above. Thus, the pressure and density of air decrease at higher altitudes. That's why a helium balloon rises: the pressure on the underside of the balloon is greater than the pressure on the top.

Air pressure falls with altitude *exponentially*,

(1) $\qquad p \; = \; p_o \, e^{-\left(\frac{mgh}{kT}\right)}$
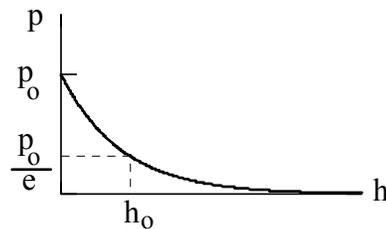
where h is the height above a level where the pressure is p_o, m is the average mass of an air molecule, k is the Boltzmann constant ($k = 1.38 \times 10^{-23} \, \mathrm{J/K}$) and T is the temperature in degree Kelvin (K). Note that both mgh and kT have the units of energy, so the exponent is dimensionless.   It is a remarkable characteristic of the exponential function that eq'n(1) is true regardless of where we set the zero of h, so long as the pressure at h = 0 is p_o. This equation (derived in the Appendix A) is not quite correct, because its derivation assumes that the atmosphere is isothermal when in fact, the temperature of the air varies considerably with altitude.

Eq'n (1) can be rewritten

(2) $\qquad\qquad\qquad p = p_o e^{-\left(h/h_0\right)}$,

where $h_o = kT/mg$ is a characteristic height , called the *scale height,* of the atmosphere. The scale height is the height increase which reduces the air pressure by a factor of $\frac{1}{e} = \frac{1}{2.718...} \cong 0.368.$ If you started at sea level (p = 1atm) and climbed a mountain with a height h_o, the pressure at the peak would be $1 \, \mathrm{atm}/e \; \cong \; 0.37 \, \mathrm{atm}.$ At an altitude of $2h_o$, the pressure would be $1 \mathrm{atm}/e^2 \cong 0.135 \, \mathrm{atm}.$

If the pressure change Δp can be measured over some small height change Δh, then the scale height h_o can be determined.

$$\frac{\Delta p}{\Delta h} \cong \left.\frac{dp}{dh}\right|_{h=0} = \left. p_o\left(-\frac{1}{h_o}\right)e^{-h/h_o}\right|_{h=0} = -\frac{p_o}{h_o}, \quad \Rightarrow \quad h_o = -p_o\frac{\Delta h}{\Delta p}. \quad (3)$$

In this experiment, you will use a differential pressure transducer to measure the change in air pressure during a elevator ride to the 10th floor of Gamow Tower ($\Delta h = 32.2$ m, according to CU records). You can feel the pressure change in you ears as you ride the elevator, so it is not surprising that it can be measured with some precision.

The principle of operation of the pressure transducer is simple. Two chambers within the device are separated by a flexible metal membrane. If there is a pressure difference between the two chambers, the membrane will flex slightly, bowing out from the high-pressure side. Small movements of the membrane are very accurately determined by measuring the capacitance between the metal membrane and a fixed metal surface nearby. The transducer output is a DC voltage ($0 \rightarrow \pm 1V$) which is proportional to the pressure difference between the two ports; when the pressures are the same, the output is 0V.
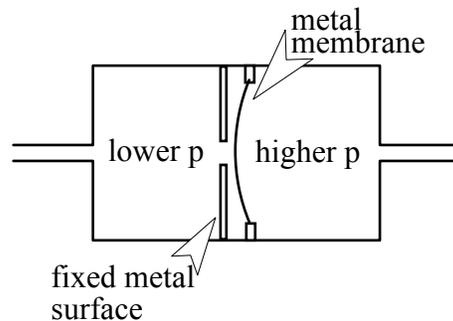
The two side chambers are labeled A and B on the instrument. When A has the higher pressure, the voltage output is positive. When A has the lower pressure, the voltage output is negative.

There is an unfortunate abundance of units for pressure. One standard atmosphere (at sea level) in various units is:

1 atm = 760 torr = 406.8 inches H_2O = 1.013 × 10^5 Pa = 1.013 bar = 1013 mbar.

A torr is a millimeter of mercury (mm Hg). A pascal (Pa) is 1 newton/meter^2, the SI unit of pressure. Here in Boulder, at an altitude of 5400 ft $\cong$ 1650 m, the pressure is about 840 mbar $\cong$ 630 torr, but it varies by ± 20 mbar, depending on the weather. You can read the current air pressure (barometric pressure) in the 1140 Lab, in cm Hg, with the mercury barometer on the wall (Ask the TA where it is).

Your first task in this lab will be to calibrate the pressure transducer by applying a known pressure difference and measuring the resulting voltage output. To do this, you will connect one side of the transducer to a sealed flask of known volume V, which can be varied

with a piston over a range ΔV. As the volume is changed, the pressure p will change according to the ideal gas law

(4)
$$pV = NkT,$$

where N is the number of molecules in the volume, k is the Boltzmann constant, and T is the temperature in $^{\circ}$K. (Air is not an ideal gas, but it is close enough for our purposes.)

For a fixed number of N of molecules, the pressure change Δp resulting from a volume change ΔV depends on whether the change is isothermal ($\Delta T = 0$) or adiabatic (no heat added or lost, $\Delta Q = 0$). The relationships between Δp and ΔV, for the two cases, are:

(5)
$$\frac{\Delta p}{p} = -\frac{\Delta V}{V} \qquad \text{(isothermal, } \Delta T=0\text{)}$$

(6)
$$\frac{\Delta p}{p} = -\frac{4}{3}\frac{\Delta V}{V} \qquad \text{(adiabatic, } \Delta Q=0\text{)}$$

The first equation (5) can be quickly derived from the ideal gas law by assuming that T and N are constant. The second equation (6) is not true for a monatomic gas, but is true for a diatomic gas, such as air; equation(6) is derived in the Appendix B.

In Part I of this experiment, the pressure change will be nearly isothermal, so we will use equation (5).

Procedure

Part 1. Calibration of the pressure transducer

In this part, you will determine the calibration constant C of the pressure transducer, which is the pressure difference per voltage output $C = \Delta p/\Delta \text{volts}$. One side of the transducer will be left open to air and the other side connected to a glass Erlenmeyer flask and a plastic syringe, as shown below.
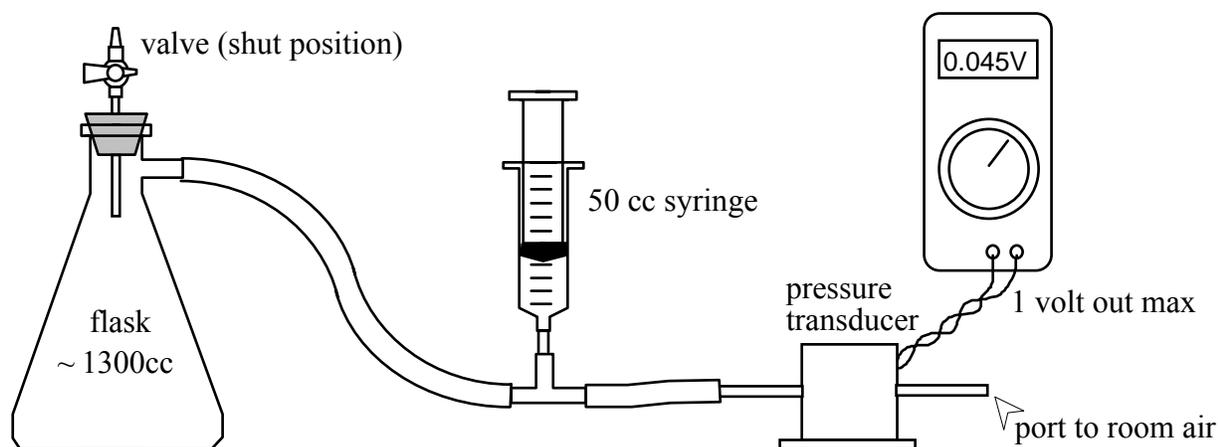
The valve on the flask can be opened to allow the pressures to equilibrate or closed to vary the pressure in the flask. The volume of the (flask + connecting hoses+ syringe with plunger at 30cc mark) is approximately 1300 cc. The precise volume is marked on the flask.

Turn the transducer on by plugging in the wires to its batteries. Open the system to room air with the valve, and check that the voltage output is close to 0.000V. Record any offset from zero. With the flask open to room air, set the syringe plunger at some point near the middle of its range, then isolate the system (flask+hose+syringe) from the room by closing the valve. When you change the volume of the enclosed air by pulling or pushing on the syringe, the piston does work on the enclosed air, resulting in a temperature change of the air. However, after about 30 seconds, the temperature of the air inside the flask will equilibrate with outside temperature, due to thermal conduction through the glass. Consequently, the final pressure change is isothermal, although the initial pressure change is somewhat adiabatic. Because of these temperature changes, you will observe that, immediately after you change the volume, the magnitude of the output voltage will reach a maximum, but then it will fall as the temperature equilibrates, reaching a stable value after less than a minute.

Pushing the syringe plunger in decreases the volume ($\Delta V < 0$); pulling it out increases the volume. Change the volume over the range -15 cc to +15 cc in $\approx$5 cc steps, and record the equilibrium voltage output for each volume. Using eq'n (5), convert the volume changes (Δvolume) to pressure changes (Δp), and plot the transducer output in volts vs. the pressure change Δp in torr. To use eq'n(5), you will need the volume V (which is marked on the flask) and the room pressure p (from the mercury barometer – remember to convert from cm Hg to torr). Your measured Δp's for positive ΔV's should be nearly the same magnitude as those for the negative ΔV's. If they are not, it is probably due to temperature drift during your measurements. Repeat until you get consistent results.

From your data, determine the calibration factor $C = \Delta$pressure $/ \Delta$volts (torr/volt) of the transducer. Do this by computing $C = \Delta p / \Delta$volts (the change from zero) for each of your 6 data points and averaging. Also, compute the standard deviation of the mean of your values.

$$(7) \qquad \overline{C} = \frac{1}{N}\sum_i C_i, \qquad \sigma = \sqrt{\frac{\sum_i \left(C_i - \overline{C}\right)^2}{N-1}}, \qquad \sigma_{mean} = \frac{\sigma}{\sqrt{N}}.$$

[In your report, be careful not to use the symbol ΔV, unless it is clear whether you mean $\Delta(\text{volume})$ or $\Delta(\text{volts})$. Check with your TA that you have approximately the right value for the calibration constant C, before proceeding to Part 2.]

Part 2. Change of pressure with temperature.

According to the ideal gas law (4), if the volume V and number of molecules N are fixed, then the pressure p is proportional to the temperature T, and the fractional change in temperature is equal to the fractional change in pressure

$$(8) \qquad\qquad \frac{\Delta T}{T} = \frac{\Delta p}{p}, \quad (\text{T in Kelvin}).$$

After opening the valve on the flask to equilibrate the pressures inside and out, close the valve to seal the system. Then warm the flask by placing your hands around it for about 20 seconds. You will see the output voltage start to change after only a few seconds. Record how long you had your hands on the flask, and record the total change in output voltage. From these data, compute the rise in pressure and the rise in temperature of the air in the flask.

Part 3. Change of pressure with altitude

To prepare for an elevator ride, disconnect the hose from the glass flask, transfer the transducer, batteries, voltmeter, etc. to the cart , and attach the hose to the thermos bottle on the cart. The thermos bottle has excellent thermal insulation, insuring that the temperature of the air inside will not vary significantly as you transport the apparatus around the building. With the valve open, depress the plunger on the syringe completely to empty the syringe of air. Leave the valve open so that the pressures on both sides of the transducer are equal. Go to the Gamow Tower elevator. (To get there, exit the 1140 lab, turn left and go down the corridor, through the double doors. The elevator is by the vending machines.)
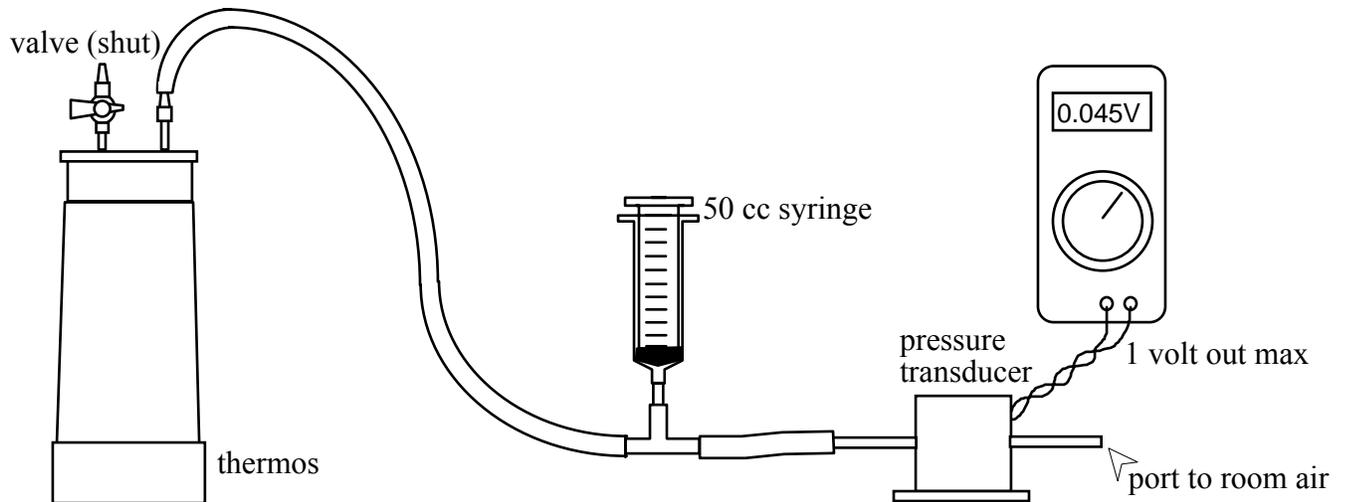
Just before getting on the elevator with your apparatus, seal the system and note the voltage output (it should be very close to 0V). Ride to the 10th floor, record the voltage there, then return to the basement and record the final voltage. When you return to the basement, the final voltage should return to close to the initial value. If your final voltage and initial voltage in the basement differ by more that about 0.020 V, something went wrong – try the trip again. The quicker you can make the trip, the better.

Take the average voltage change from your trip up and trip down and compute the pressure change between the basement and the 10th floor. The height change is $\Delta h = 32.2 \text{ m}$. With your data and eq'n (3), compute the scale height h_o of the atmosphere. Calculate the uncertainty Δh_o.

Appendix A: Derivation of $p = p_o e^{-\left(\frac{mgh}{kT}\right)}$

A layer of air has weight; it is held up because the pressure below is greater than the pressure above. Consider a layer of air of thickness Δh, area A, and density ρ; its volume is $A \cdot \Delta h$ and its weight is

(A.1) $\qquad\qquad w = mg = \rho \cdot A \cdot \Delta h \cdot g$
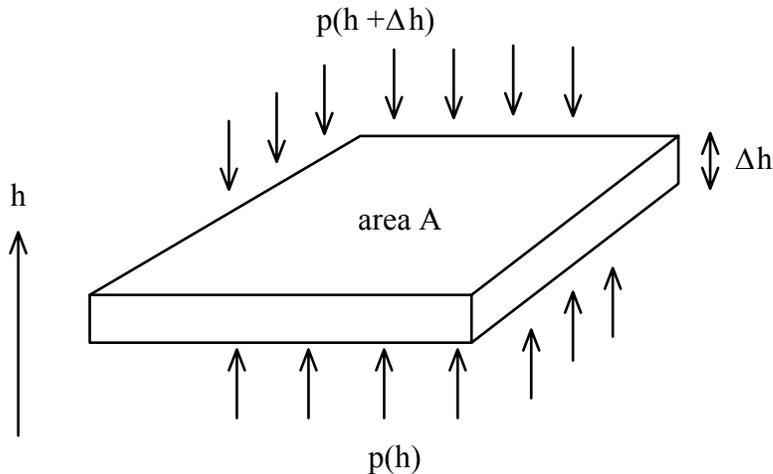
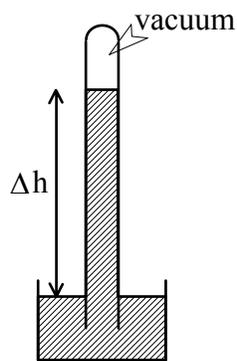This weight is supported by the *net* force on the layer due to both the pressure above and the pressure below:

(A.2) $p(h) \cdot A - p(h + \Delta h) \cdot A \;=\; \rho \cdot A \cdot \Delta h \cdot g$,

or,

(A.3) $p(h + \Delta h) - p(h) \;=\; -\rho \cdot \Delta h \cdot g \quad \Rightarrow \quad \dfrac{dp}{dh} \;=\; -\rho g$.

<u>If</u> the density is constant, we have

(A.4) $|\Delta p| = \rho\, g\, |\Delta h|$.

This equation is the principle of operation of barometers. The pressure due to the weight of a column of fluid (usually mercury) with density ρ and height Δh, is balanced by atmospheric pressure. From (A.4), one can show that the height of a column of mercury in a mercury barometer at STP (standard temperature and pressure) is 760 mm.

The density of air is not constant, however; it varies with pressure. From the ideal gas law,

(A.5) $p \;=\; \dfrac{N}{V}\, kT \;=\; \dfrac{Nm}{V}\, \dfrac{kT}{m} \;=\; \rho\, \dfrac{kT}{m}$,

where m is the average mass of an air molecule and ρ is the mass density of air. Hence,

(A.6) $\rho \;=\; \left(\dfrac{m}{kT} \right) p$.

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Inserting this into (A.3) yields

(A.7) $$\frac{dp}{dh} = -\left(\frac{mg}{kT}\right) p$$

The solution of this differential equation is

(A.8) $$p = p_o\, e^{-\left(\frac{mgh}{kT}\right)}.$$

This equation is only approximately true because we have made two assumptions which are not strictly correct: air is an ideal gas and the atmosphere is isothermal.

Appendix B: Derivation of pressure change during an adiabatic volume change.

The pressure change due to an adiabatic volume change depends on whether the gas in question is monatomic, like helium or neon, or diatomic, like the main components or air, oxygen and nitrogen. Assuming that the gas container is thermally isolated from the outside world, then no heat is exchanged during the volume change. However, the piston does work on the gas as the volume changes, so energy, in the form of work, is exchanged with the outside and the temperature of the gas varies.

We first derive the temperature change ΔT. We will need the First Law of Thermodynamics:

(B.1) $$\underbrace{\Delta U}_{\text{energy added}} = \underbrace{-p\,\Delta V}_{\text{work done on the system}} + \underbrace{\Delta Q}_{\text{heat added}}$$

and an expression for the total internal energy of a gas of N atoms or molecules. For the case of a *monoatomic* ideal gas:

(B.2) $$U = N \times KE \text{ per atom} = \tfrac{3}{2}NkT.$$

While for a diatomic gas,

(B.3) $$U = N \times (KE + \text{rotational energy}) \text{ per molecule} = 3NkT.$$

Equations (B.2) and (B.3) can both be written as $U = bNkT$, where b is either 3/2 or 3. Assuming $\Delta Q = 0$, (B.1), (B.2), and (B.3) yields

(B.4) $$bNk\,\Delta T = -p\,\Delta V \quad \Rightarrow \quad \Delta T = -\frac{1}{b}\frac{p}{Nk}\Delta V = -\frac{1}{b}T\frac{\Delta V}{V}.$$

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So, for an adiabatic change of volume, we have

(B.5) $$\frac{\Delta T}{T} = -\frac{1}{b}\frac{\Delta V}{V}.$$

Now we can compute the pressure change. Starting from the ideal gas law, $p = \dfrac{NkT}{V}$, and differentiating, we have

(B.6) $$\Delta p = \frac{Nk}{V}\Delta T - \frac{NkT}{V^2}\Delta V = \frac{p}{T}\Delta T - \frac{p}{V}\Delta V.$$

Substituting (B.5) into (B.6) gives

(B.7) $$\Delta p = \frac{p}{T}\left(-\frac{1}{b}T\frac{\Delta V}{V}\right) - \frac{p}{V}\Delta V = -\left(1+\frac{1}{b}\right)p\frac{\Delta V}{V} = -\left(\frac{5}{3}\text{ or }\frac{4}{3}\right)p\frac{\Delta V}{V},$$

The coefficient is 5/3 for a monatomic gas and 4/3 for a diatomic gas.

PreLab Questions:

1. The ideal gas law states $pV = NkT$. If N and T are fixed, then pressure is inversely proportional to volume: $p = \dfrac{C}{V}$, where C is a constant. Consider a large volume V with fixed N and T. If V changes by a small amount ΔV, what is Δp, the change in p? (Hint: remember calculus: if $f = f(x)$, then $\delta f = \dfrac{df}{dx}\delta x$.) Use this result to prove eq'n (5).

2. On planet X (not earth), the pressure at sea level is 300 torr. The pressure falls by 1.0 torr when the height is increased by 60 m. What is the scale height h_o on planet X?

3. What is the average air pressure (barometric pressure) in Boulder in units of mm Hg?

4. Compute the scale height of the atmosphere, h_o. Assume that the temperature of the atmosphere is 300K and the average mass of an air molecule is $m = 4.78 \times 10^{-23}$ gram.
Give the answer in both meters and miles. (Hint: Be sure to use a consistent set of units in your calculation.)

5. Assume the scale height of the atmosphere is $h_o = 4.00$ km (it is actually larger than this). By what factor does the pressure fall as your height is increased from 2 km to 6 km? From 24 km to 32 km? [Hint: you can answer this question with a very simple calculation (simple enough to do

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in your head). If you answer this question with a very complicated calculation, you will receive only partial credit.]

6. In this problem, you can assume that the scale height of the earth's atmosphere is about 8 km .You return from a hike and discover that your empty plastic 1 liter water bottle has a dent on one side. When you take off the lid, air rushes in and the dent disappears. If you hiked down from a pressure of 560 torr to a pressure of 620 torr, by approximately how much did the volume of the 1 liter (1000 cc) bottle decrease? (Assume no temperature change and that the plastic does not resist the volume change.)

7. In this experiment, when you use eq'n (3) to compute h_o from your measured values of Δh and ΔV, how do you know the value of p_o?

8. Use eq'n (A.4) to show that the height of a column of mercury in a barometer is 760mm at STP. The density of mercury is 13.6 grams/cm^3. (Hint: In a barometer, Δp is 1 atmosphere, but you have to use a *consistent* system of units.)

9. Prove eq'n (8). Start with the ideal gas law and the assumption of constant N and V.

10. In part 1, you are to plot the voltage output of the pressure transducer vs. the pressure change. Sketch what this graph should look like. How is this graph related to the calibration constant C of the pressure transducer?

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