Lab M6: The Doppler Effect

Introduction

The purpose in this lab is to teach the basic properties of waves (amplitude, frequency, wavelength, and speed) using the Doppler effect. This effect causes the frequency of sound waves to be higher for a source that is approaching you and lower for a source that is moving further away.

Sound is a pressure wave in air. When we hear a sound, we are sensing a small variation in the pressure of the air near our ear. The speed of a sound wave in air, v_{sound}, is about 340m/s; sound travels 1 mile in about 5 seconds. This speed depends only on the properties of the air (temperature, composition, etc.) , not on the frequency or wavelength of the wave and not on the motion of the source or receiver.

Consider a sinusoidal sound wave in air with frequency f and wavelength λ. The speed v_{sound} is related to f and λ by

(1) $$v_{sound} = f\,\lambda.$$

To see where this relation comes from, think:

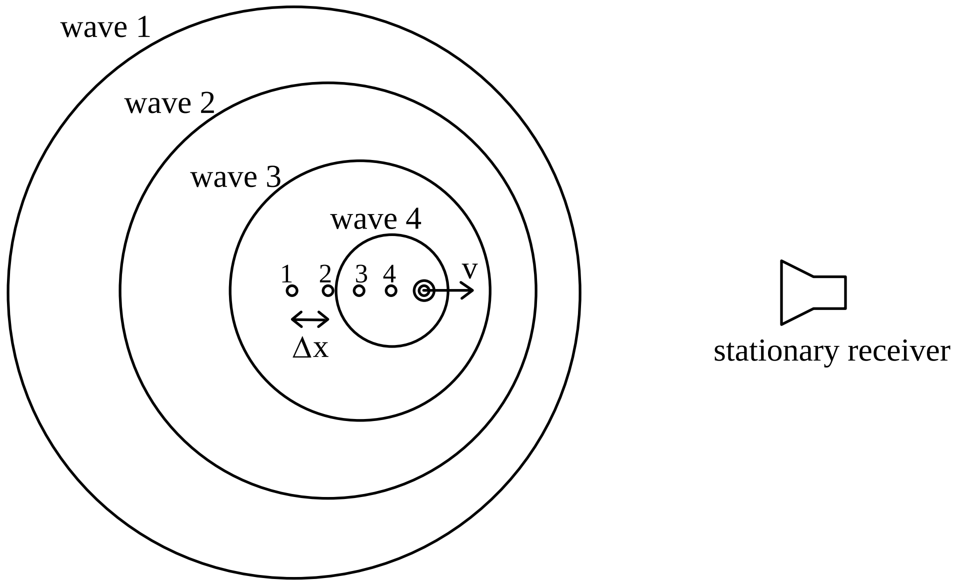
$$\text{speed} = \frac{\text{change in distance}}{\text{change in time}}.$$

The time it takes for one wavelength of the sound to pass by is the period T, so $v = \lambda/T$. But $f = 1/T$ so $v = f\,\lambda$. Note that as f increases, λ goes down, but the speed v stays the same. The frequency range of human hearing is about 20 Hz to 20,000 Hz. (The upper end drops as we age; for people over 60, it is 8 -13 kHz, while dogs can hear up to about 35 kHz.)

When the siren from an ambulance passes by a listener, the listener reports a change in the pitch of the siren. This is the Doppler effect for sound waves. A source of sound moving away from the receiver (listener) has a lower apparent frequency (tone or pitch), while a source moving toward the receiver has a higher frequency. The frequency shift is approximately proportional to the relative speed of the source and receiver.

To understand the Doppler effect, consider a stationary receiver detecting the sound from a moving source, a source which is emitting a sound wave with constant frequency f_o. Let us consider the case in which the source is moving directly toward the stationary receiver with a speed v. In this context, “stationary” and “moving” mean at rest or not with respect to the air which carries the sound.

The diagram below shows a series of successive wavefronts emitted when the source was at positions 1, 2, 3, 4, and 5. . The time between successive wavefronts emitted by the source is the period $T_o = \frac{1}{f_o}$, and during the time T_o, the source moves a distance $\Delta x = v \cdot T_o = v \cdot \frac{1}{f_o}$.



moving source, speed v, frequency f_o

If the source were not moving, the spherical wavefronts would be concentric, centered on the stationary source, and the distance between successive wavefronts would be the wavelength λ_o. The speed of sound v_{sound}, the wavelength λ_o , the frequency f_o, and period T_o are related by equation (1)

$$v_{sound} = \frac{\lambda_o}{T_o} = \lambda_o f_o .$$

Because the source is moving, the wavefronts are crowded together on the side near the receiver, and the distance between wavefronts, on the receiver side, is

(2)
$$\begin{aligned} \lambda' &= \lambda_o - \Delta x = \lambda_o - v \cdot T_o = \lambda_o - \frac{v}{f_o} \\ &= \lambda_o - \lambda_o \frac{v}{v_{sound}} = \lambda_o \left(1 - \frac{v}{v_{sound}}\right). \end{aligned}$$

These wavefronts strike the receiver at intervals of T' seconds with a frequency $f' = \frac{1}{T'}$,

which is related to v_{sound} and λ' by [equation (1) again]

$$v_{sound} = \frac{\lambda'}{T'} = \lambda' f' .$$

Solving for f', we have

(3) $$f' = \frac{v_{sound}}{\lambda'} = \frac{v_{sound}}{\lambda_o\left(1-\frac{v}{v_{sound}}\right)} = \frac{f_o}{\left(1-\frac{v}{v_{sound}}\right)}$$ (receiver stationary, source approaching).

The frequency is shifted up and the receiver hears a higher pitch. With some algebra, one can show that the frequency shift $\Delta f = f' - f_o$ is given by

(4) $$\Delta f = f_o \cdot \left(\frac{\frac{v}{v_{sound}}}{1-\frac{v}{v_{sound}}}\right) ,$$

which, if $v << v_{sound}$, is approximately

(5) $$\Delta f \cong f_o \cdot \frac{v}{v_{sound}} .$$

From eq'n (3), solving for v_{sound}, we have

(6) $$v_{sound} = v\left(\frac{f'}{f' - f_o}\right) = v\left(\frac{f'}{\Delta f}\right).$$

Thus, one can determine v_{sound} from measurements of the frequency shift and the speed of the source.

If the source is receding from, rather than approaching the stationary receiver, then the (–) sign in eq'n (4) is replaced with a (+) sign, and the frequency is shifted down. If the source is stationary, and the receiver is moving, then a new derivation is required and one obtains a different formula (not shown here).

Radar guns use the Doppler effect, but with electromagnetic waves, rather than sound waves. The policeman's radar gun acts as both a transmitter and receiver. It emits a radar signal of known frequency, which is reflected from a moving car, and the Doppler-shifted reflected signal is detected.

Experiment

The source is an ultrasonic speaker, which emits a tone at about 40 kHz – well above the range of human hearing. The source rides on an air track glider, whose speed is determined with a pair of photogates and a timer. An ultrasonic receiver detects the sound from the source and the frequency is displayed on a high-precision frequency counter. Both the transmitter and the receiver signals are displayed on the oscilloscope. The experiment is to measure the frequency shift as a function of the speed of the source, and, using these data, to compute the speed of sound.

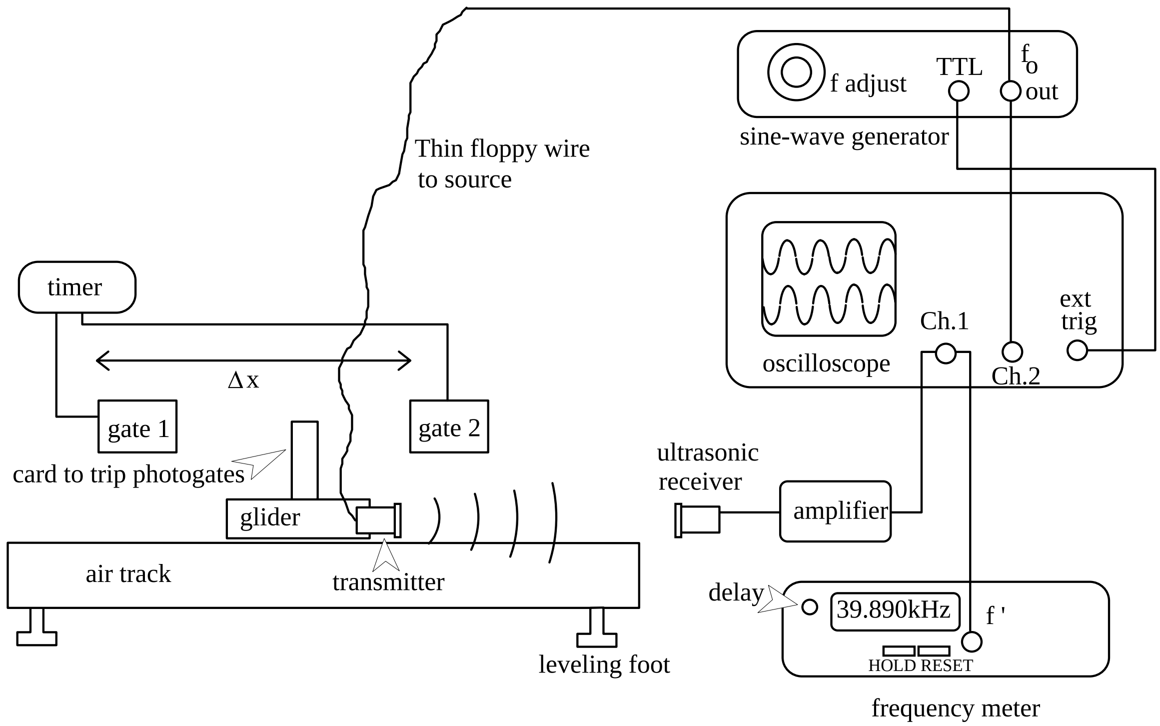
A schematic of the experimental apparatus is shown below. Begin by leveling the air table. After turning on the air supply, place the glider midway between the photogates, and adjust the height of the leveling foot until the glider remains stationary. Make sure that the blocking card attached to the glider interrupts the photogates. The timer starts when the card interrupts gate 1, stops when the card interrupts gate 2, and displays the time interval Δt. The resolution of the timer can be set to either 1 or 0.1 msec resolution, but with 0.1msec resolution, the timer overflows at only 2 sec; use 1 msec resolution to avoid overflow.

By measuring the distance Δx between the two photogates, the speed v of the glider can be computed from $v = \frac{\Delta x}{\Delta t}$. Make sure that nothing interferes with thin wires leading to the source, as the glider travels back and forth on the air track.

On the oscilloscope, observe both the signal to the source (on channel 2) and the signal from the receiver (on channel 1). The ultrasonic transducers used for the source and receiver resonate at a fixed frequency of around 40 kHz; they do not function well at frequencies far from 40 kHz. With the source stationary, turn the fine adjust knob on the sine-wave generator to tune the frequency of the source for maximum signal on the receiver. After allowing the sine-wave generator several minutes to warm up and stabilize, record the frequency f_o of the source with the frequency meter. With the glider stationary, there is no Doppler shift, and the frequency of the source and receiver are identical. Hold the glider stationary and read f_o from the receiver frequency meter, which has a 1 Hz resolution. Check f_o frequently; if it is drifting you will have to record it each time you take data.

Make sure that the receiver is directly in front of the transmitter, at the same height, etc. so that the source moves directly toward the receiver.

The TTL output of the sine-wave generator is used to externally trigger the oscilloscope. This ensures that there is always a stable display on the oscilloscope.



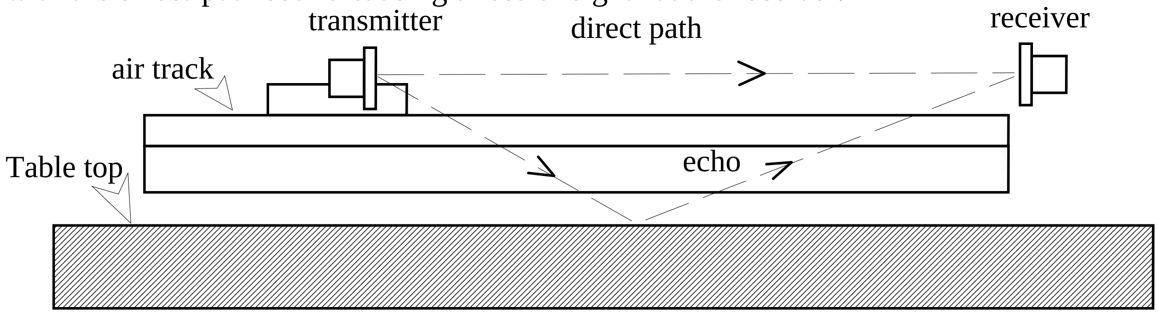
Procedure

Part 1. Speed of sound from the measured Doppler shift

Your task is to measure the frequency f' at the receiver and hence the frequency shift $\Delta f = f' - f_o$, as a function of the speed of the source. From these data, you will compute the speed of sound, as shown below. Reset the timer to zero and push the glider toward the receiver so that it passes through gates 1 and 2 at a uniform speed. Record the frequency f' on the frequency meter and the photogate time interval Δt. Repeat this procedure for several (at least 10) trials, varying the speed of the glider over as large a range as possible. You cannot let the speed of the glider go much faster than 1 m/sec (which is pretty fast!) or the frequency counter won't have time to record the frequency.

The frequency meter works by counting cycles for one second (frequency in Hz = number of cycles in one second). There is a little red light which is on while the meter is counting. To prepare the frequency meter to take a reading, make sure the FREQ button is depressed, turn the DELAY knob to the HOLD position, and then press the HOLD button. When you are ready to record the frequency, press the RESET button. As soon as you press RESET, the meter will count cycles (the red light will come on) for 1 sec and then the frequency will be displayed (in kilohertz, kHz). When taking measurements at faster speeds, you should press the RESET button immediately after pushing the glider so that the 1 second count time is up before the glider reaches the end of the track.

On the table near the track there is some sound-absorbing foam material which serves the important function of preventing reflected sound (echoes) from reaching the receiver. Make sure the foam covers the side of the track. If the sound absorbers were not present, then the receiver would receive not only the direct-path sound from the transmitter but also various reflected-path echoes. The echoes can interfere destructively with the direct-path sound causing a loss of signal at the receiver.



If the receiver signal drops below about 0.2V peak-to-peak, then the frequency counter cannot count correctly and the displayed frequency will be too low. Check that the receiver is getting adequate signal by observing the oscilloscope output as you slide the transmitter along the whole length of the track. Stop the transmitter at any points along the track where the receiver signal is unusually small, and check that the frequency counter is still displaying f_o.

Plot your measured frequency shift $\Delta f = f' - f_o$ vs. the speed v of the source. You should get a very straight line, as predicted by eq'n (5). From the appearance of the graph, decide whether any of the data points should be thrown out. (If one point deviates substantially from the others, it is usually an indication that a number was recorded incorrectly.) For each of the good data points, compute the speed of sound from eq'n (6). Using your several values of v_{sound} , compute the mean, the standard deviation, and the standard deviation of the mean. (To avoid subscript chaos, we write "vs" for v_{sound} below.)

$$\overline{vs} = \frac{1}{N}\sum_i vs_i \qquad \sigma = \sqrt{\frac{\sum_i (vs_i - \overline{vs})^2}{N-1}} \qquad \sigma_{mean} = \frac{\sigma}{\sqrt{N}}$$

Part 2. Comparison with the known speed of sound

The speed of sound in air depends on the temperature according to

$$v(m/s) = 331.5 + 0.607 \cdot T \tag{7}$$

where the temperature T is in degrees Celsius. This expression is only accurate for temperatures near room temperature. (You can read the temperature from the big dial thermometer on the wall in the lab). Compare this theoretical v_{theory} computed from eqn(7) with your measured v_{meas} from Part 1 .

Part 3. Calculation of the wavelength of the ultrasonic wave

From the known frequency f_o and your measured v_{sound} from Part 1, compute the wavelength λ_o of the sound. Also compute the uncertainty $\delta\lambda_o$.

Part 4. Interferometry and wavelength

In an interferometer, a signal that has taken a direct path from a source to a receiver is compared with a signal that has taken an alternate path. On channel 1 of the oscilloscope is the electrical signal directly to the transmitter. On channel 2 is the signal from the receiver that “hears” the sound after passing through about a meter of air. If this air path is a whole number (N) of wavelengths, these two signals will be in phase and will add. If the air path is N + 1/2 wavelengths, the signals will be out of phase and will tend to cancel. The subtraction will not give exactly zero unless the two signals are exactly the same amplitude. The time for the *electrical* signals to travel can be ignored in comparison with the time for the sound to travel through air.

Adjust the Channel 1 and Channel 2 volts/division knobs on the oscilloscope so that the two signals are nearly three divisions in height (peak-to-peak). (You may have to fiddle with the little "cal" knobs in the middle of the bigger knobs.) Above the channel 2 knob there is an ADD-ALT-CHOP switch that you should move to the ADD position. Now the oscilloscope displays the sum of the direct signal and the signal that has gone through the air. Move the glider with your hand and notice the change in amplitude of the summed signals. Switch briefly back to the ALT or CHOP position to see the signals move from in phase to out of phase. Return to the ADD position and measure with a ruler the distance that you must move the glider to pass through 10 to 20 maxima or minima. Divide this distance by the number of maxima or minima to determine the wavelength from interferometry λ_{int}. What is $\delta\lambda_{int}$? How does it compare with the wavelength λ_o that you calculated in Part 3? Calculate the difference between the two measurements $\Delta\lambda = \lambda_{int} - \lambda_o$ and $\delta\Delta\lambda = [\delta\lambda_{int}^2 + \delta\lambda_o^2]^{1/2}$.

PreLab Questions:

1. What is the approximate frequency of the ultrasonic transmitter used in this experiment? What is the wavelength of sound at that frequency?

2. **(Counts as two questions)** Four things are measured directly in this lab: Δx, the distance between the photogates, f_0, the frequency of the source, Δt_i, the time for the glider to travel between the photogates (trial i), and f_i', the frequency at the receiver (trial i). Using Mathcad statements, show how to compute v_i, the speed of the glider (trial i), and v_sound$_i$, the speed of sound (trial i), from these measured quantities.

3. Show that eq'n (6) follows from eq'n (3).

4. Eq'n (4) is an exact expression for Δf, while eq'n (5) is only approximate. Using $v_{sound} = 345$ m / s, v = 0.30 m/s, and f_o = 40.0 kHz, compute Δf using both equations (4) and (5). By how much do the two values differ? What is the fractional difference? (Since we want to compare the two values, display several digits for the two Δf 's, rather than rounding to significant figures.)

5. Sketch the graph of Δf vs. v. What is the approximate slope of this graph? Give a algebraic expression and a numerical value for the slope.

6. How is it determined whether the air track is level?

7. Assume that the values of v_{sound}, δv_{sound}, f_o, and δf_o are known. Show how to compute $\delta\lambda_o$, the uncertainty in the wavelength.

8. On Jan. 17, 1930, it was -33F (-36C) in Boulder, with an uncertainty of ±1 ^{o}C. What was the speed of sound outside on that day (include the uncertainty in the speed of sound)?

9. The air track in this experiment is 2 meters long. If the glider goes faster than 2 m/s, this experiment won't work, at least not with the existing equipment. Why not? [Hint: how is f' measured?]

10. When the source is *receding* from a stationary receiver and the speed of the source approaches the speed of sound, what happens to the Doppler shift Δf? What happens to Δf when the source is *approaching* a stationary receiver and its speed approaches the speed of sound.