Lab E4: Capacitors and the RC Circuit

Capacitors are among the most-commonly used circuit elements in modern electronic devices. Your calculator and cell phone contain dozens or hundreds of them. Any two electrical conductors brought near each other form a capacitor. If a charge Q is transferred from one conductor to the other (so that one has a charge +Q and the other has charge -Q), then there will be a voltage difference V, which is proportional to the charge Q. The constant ratio $\dfrac{Q}{V}$ is called the capacitance C.

(1)
$$C = \frac{Q}{V}, \qquad V = \frac{Q}{C}.$$

[To see that V must be proportional to Q, recall that the electric field $\vec{E}$ is proportional to the charge ($\vec{E} \propto Q$), and the voltage is proportional to $\vec{E}$ ($V \propto \vec{E}$), hence, $V \propto Q$.]

A parallel-plate capacitor, formed by two flat plates of area A, separated by a distance d, has a capacitance given by

(2)
$$C = \varepsilon_o \frac{A}{d},$$

where ε_o is the permittivity constant, which has the value $\varepsilon_o = 8.85 \times 10^{-12}$ in SI units. Eq'n (2) applies when the plates are separated by air or vacuum. When there is a dielectric medium such as plastic or glass between the plates, the capacitance is greater than predicted by (2) and the correct formula is

(3)
$$C = \kappa \varepsilon_o \frac{A}{d}.$$

κ (κ = Greek letter kappa) is a dimensionless number called the dielectric constant. The value of κ depends on the dielectric medium, but it is always greater than one ($\kappa > 1$). The dielectric constants of various media are ..

Material	κ
nylon	3.4
oil	4.5
Pyrex	4.7
silicon	12
titania ceramic	130
strontium titanate	310
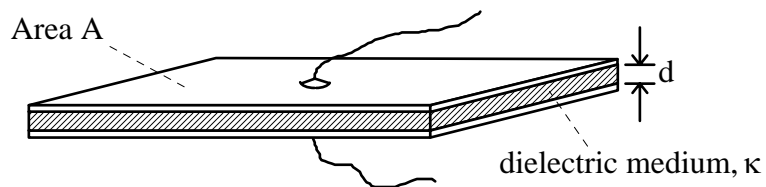
A parallel-plate capacitor

The SI unit of capacitance is the farad (F): $1 \text{ farad} = 1 \text{ coulomb} / \text{volt}$. A farad is an immense capacitance; a typical capacitor in an electronic circuit has a capacitance of

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micro-farads ($1\mu F = 10^{-6}F$) or nano-farads ($1nF = 10^{-9}F$). Even a μF capacitor is regarded as a rather large capacitance and making such a capacitor in a small package is quite a technological feat. Consider trying to a make a $1\mu F$ capacitor with two square plates separated by a 1mm (10^{-3} m) air gap. The area A needed is

$$A \quad = \quad \frac{C\,d}{\varepsilon_o} \quad \approx \quad \frac{10^{-6} \cdot 10^{-3}}{10^{-11}} \quad = \quad 100\,m^2.$$

These plates would be about 30 ft on a side! Capacitors with large capacitance and small physical size are produced by making d extremely small ($d \approx 10^{-6} - 10^{-8}\,m$) and κ large ($\kappa \approx 100$).

A capacitor can be thought of as a device which stores charge. If the two sides of a capacitor are connected across a battery of voltage V_o and then disconnected from the battery, each plate of the capacitor will carry a charge of magnitude $Q_o = C\,V_o$ (one side has $+Q_o$, the other has $-Q_o$). The larger the capacitance, the larger the charge carried. If the two sides of this charged capacitor are then connected by a wire, there will be a large, brief current through the wire as the capacitor discharges. If we use a large resistance R, instead of a wire, to connect the sides of the charged capacitor, the capacitor will discharge much more slowly. The charge and the voltage on the capacitor will decrease *exponentially* in time, with a *time constant* $\tau = RC$ (τ = Greek letter tau).

Consider the simple RC circuit shown below. The capacitor C can be connected to the battery V_o with switch S1 or to the resistor R with switch S2.
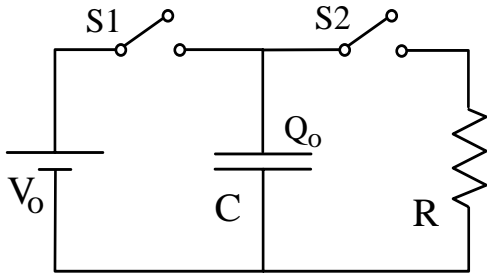
If switch S1 is closed and then re-opened, the capacitor will be charged to a voltage V_o and will carry a charge $Q_o = C\,V_o$.
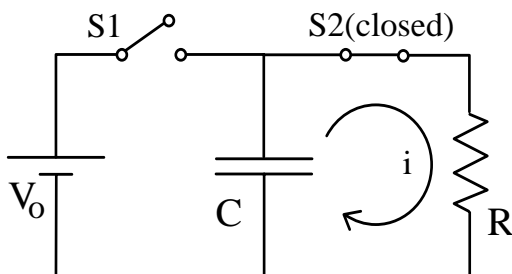
If we then close switch S2, a current i will flow through the resistor as the capacitor discharges. The current i, the charge q remaining on the capacitor, and the voltage v across the capacitor and resistor are related by..

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$$v = iR = \frac{q}{C}, \qquad i = -\frac{dq}{dt},$$

The minus sign in the second equation indicates that the charge q is getting smaller as the current i flows. Combining these two equations yields the differential equation

(4)
$$\frac{dq}{dt} = -\frac{q}{RC} = -\frac{q}{\tau},$$

where we have written $\tau = RC$. Notice that $\tau = RC$ has the units of time.

With the initial condition $q(t=0) = Q_o$, the solution to this differential equation is

(5)
$$q(t) = Q_o \, e^{-(t/\tau)}.$$

The voltage v(t) is given by

(6)
$$v(t) = \frac{q}{C} = \frac{Q_o}{C} e^{-t/\tau} = V_o \, e^{-t/\tau}.$$

Taking the natural log of both sides of (6) yields

(7)
$$\ln(v) = -\frac{t}{\tau} + \ln(V_o).$$

Hence, a plot of ln(v) vs. t should be a straight line with slope = $-1/\tau$.

Experiment

Part I. Capacitance of parallel plates

In this section, you will measure the capacitance of a parallel-plate capacitor as a function of the plate separation. Then, using equation (2), you will compute the permittivity constant ε_o. This experiment must be performed on a wooden table because the presence of large, electrically conducting objects (such as metal tables) would affect the delicate capacitance measurement.

Here, the capacitor consists of two large aluminum plates held apart by nylon plastic washers. Begin by measuring the dimensions of the plates and compute the area A. Then, check the zero of the capacitance meter by disconnecting any wires to the meter, setting the range to 200 pF (the most sensitive setting), and turning the zero knob very gently until the reading is within 1pF of zero. The zero knob is *extremely* sensitive; turn it

just a little bit. When using the meter to measure capacitance, always set the range for the most sensitive setting which does not cause overflow.

Now you are ready to measure the capacitance of the aluminum plates. Place plastic washers at the four corners of the capacitor plate holder, so that the plate separation is the thickness of 1 washer. Now step back away from the plates a few feet and note the capacitance on the meter to the nearest pF = 10^{-12} F. [Your body is full of salty water, which is a good conductor, so the presence of your body can affect the reading if you are too close.]
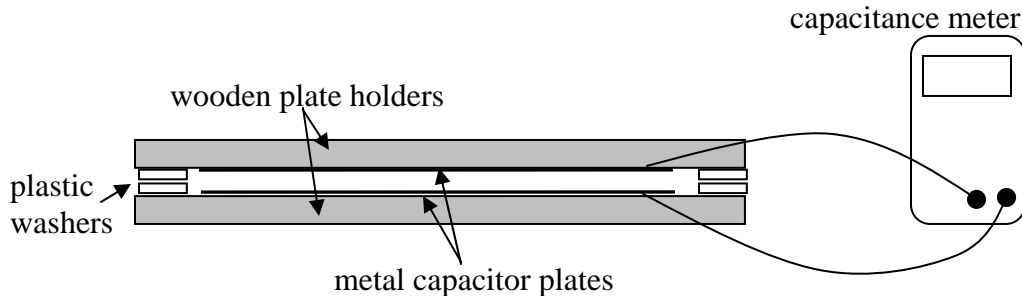
Now repeat this procedure with stacks of washers so that the plate separation equals the thickness of 2 washers, 3 washers, ... up to 6 washers.

In this measurement, the capacitance meter measures not only the capacitance between the plates, but also the *stray capacitance*, which is the capacitance between the wires + the capacitance between each wire and the opposite plate. You can measure this stray capacitance by noting the capacitance reading when the plates are both flat on the table and not too close to each other, as in the drawing to the right. The idea is to reproduce, as nearly as possible, all the measurement conditions, except without the plates over each other. Subtract the measured stray capacitance from your earlier measurements to determine the inter-plate capacitance for each of the six separations.
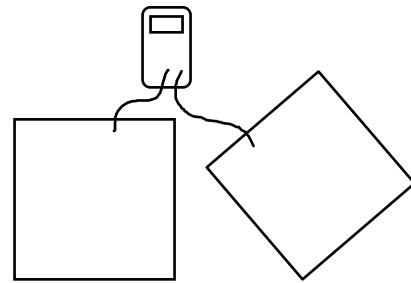
Now you must determine the average thickness of the nylon washers, in order to compute the separation of the plates for the six measurements. The washers are not perfectly identical; there is a slight variation in thickness from washer to washer. You can quickly get a very precise average thickness by stacking 30 or more washers and measuring the total thickness of the stack with the white plastic calipers. Divide your total stack height by number of washers to get the thickness of a single washer. (Use the "poker chip tray" provided to stack the washers easily.)

Plot the inter-plate capacitance C vs. (1/d). Does the graph agree with theory? Now use your data (measured values of A, d, and C) to compute ε_o for each of your six measurements, and use eq'n (8) to compute ε_o for each of your six plate separations.

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Compute the average of ε_o, as well as the standard deviation and the standard deviation of the mean. As usual, compare your result with the known value and comment on any discrepancy.

Part II. Measurement of the time constant of an RC circuit.
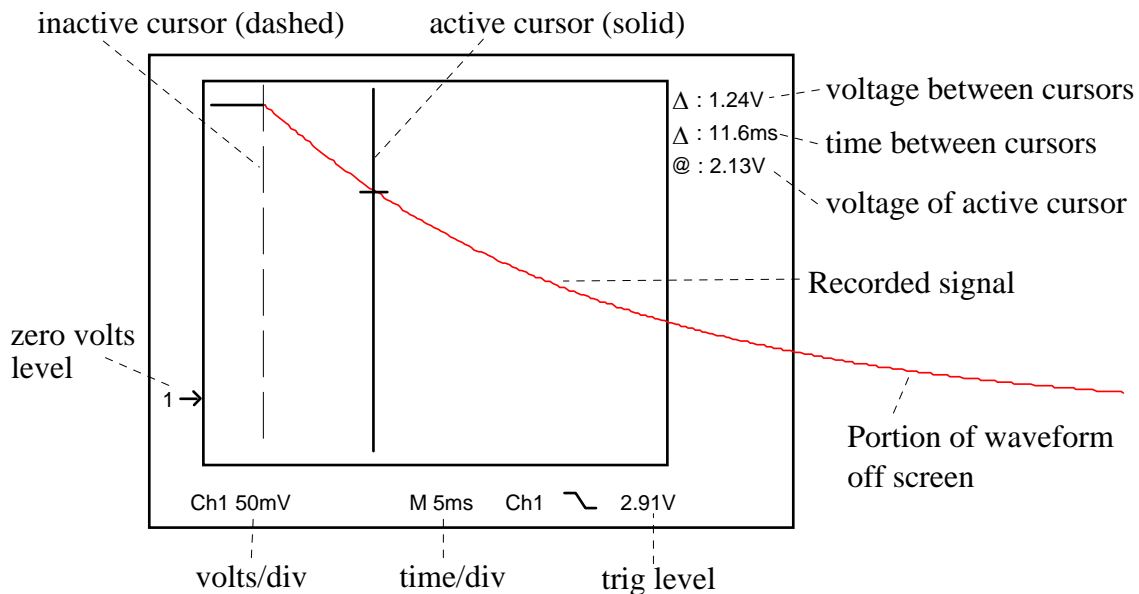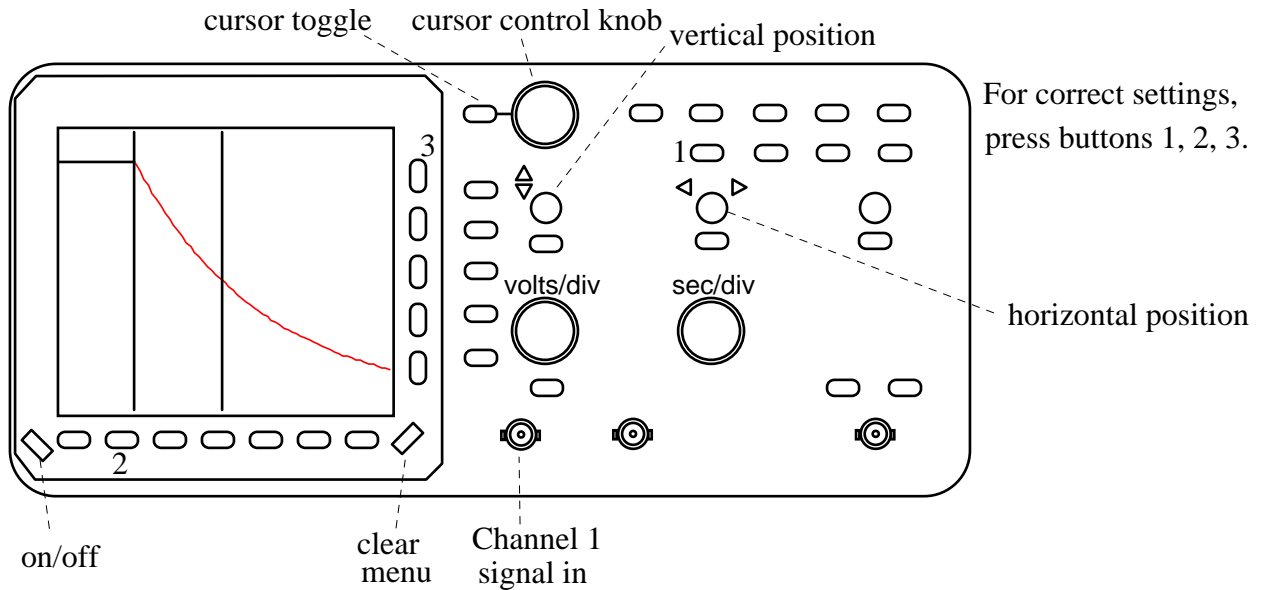
In this part, you will observe the exponential decay of the voltage on a capacitor in an RC circuit, using a sophisticated digital oscilloscope. You will measure the time constant τ of the decay and compare with the computed $\tau = RC$.

In this part, you will use a commercial resistor and capacitor (not the aluminum plate capacitor of the last section). Before assembling the circuit shown below, measure the capacitance C of the capacitor and the resistance R of the resistor using the capacitance meter and the digital multimeter. From your measured values, compute RC; it should be in the range 10 - 50 msec. Now assemble the circuit and set the power supply voltage somewhere between 3.0 and 3.5 volts.
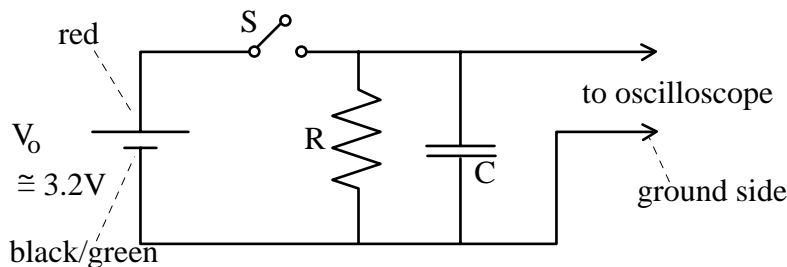
At first glance, the oscilloscope has a formidable-looking front panel. Fortunately, all the correct front-panel settings are stored in memory and can be recalled at any time with following sequence of commands:
1) Press SAVE/RECALL SETUP button on the upper right of the scope.
2) On the row of button below the screen, press RECALL SAVED SETUP.
3) On the column of buttons just to the right of the screen, press RECALL SETUP 1.

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cursor toggle   cursor control knob   vertical position

For correct settings,
press buttons 1, 2, 3.

volts/div   sec/div

horizontal position

on/off

clear
menu

Channel 1
signal in

inactive cursor (dashed)   active cursor (solid)

Δ : 1.24V   voltage between cursors

Δ : 11.6ms   time between cursors

@ : 2.13V   voltage of active cursor

Recorded signal

zero volts
level

1→

Portion of waveform
off screen

Ch1 50mV   M 5ms   Ch1   2.91V

volts/div   time/div   trig level

With the circuit properly assembled, close the switch S and then open it. The digital oscilloscope will record the brief exponential decay of the voltage across the capacitor when the switch S is opened.

The recorded signal should appear as in the diagram above. As you can see, there is a lot of information displayed on the screen. The volts/division of the vertical scale and the time/division of the horizontal scale are shown at the bottom of the screen. The zero volts position is shown with a little (1→) symbol (1 refers to Channel 1).

You can quickly read the voltage and time of any point on the recorded waveform by using the two cursors which are controlled with the cursor toggle button and the cursor control knob. (If any screen menus are displayed, press CLEAR MENU before using the cursor controls.) One cursor or the other is made "active" with the toggle button and the active cursor is moved with the control knob. The active cursor is shown as a solid vertical line; the inactive cursor is shown as a dashed vertical line. The vertical distance (Δvoltage) and the horizontal distance (Δtime) between the two cursors is shown at the upper right of the screen, along with the absolute voltage of the active cursor. The oscilloscope screen displays only a portion of the recorded waveform. You can view more of the recorded waveform with the horizontal position knob.

You want to measure time from the moment when the switch S was closed and the voltage on the capacitor began to decay. So set one of the cursors directly over the point where the exponential decay begins. Then toggle to the other cursor. By moving the other cursor, you have a readout of the time and voltage of any point on the waveform.

Record the voltage and time of 10 or more points along the exponential decay, including at least 5 or 6 points in the first part of the curve between the initial voltage V_o and $V_o/3$. Using all your data, make plots of V vs. t and ln(V) vs. t.

The plot of ln(V) vs. t should be a straight line with slope $-1/\tau$. *Using only the points between voltages V_o and $V_o/3$,* determine the slope m and the intercept b of the best fit line using the file linfit.mcd , which computes the best fit line to any x-y data. Do not use the data points at small voltages, because these have large fractional errors and will reduce the precision of the fit. The file linfit.mcd is on your PC. In Mathcad, open linfit.mcd, enter your x-y data ($x = t$, $y = \ln(V)$) and linfit computes m, b, δm, and δb. You can switch back and forth between linfit.mcd and your original file with the WINDOW menu item.

From your computed slope m and intercept b, plot the best fit line on your graph of ln(V) vs. t. From the slope $m \pm \delta m$, compute the time constant $\tau \pm \delta\tau$. Compare τ with RC, computed earlier.

Prelab Questions.
1. Demonstrate that eq'n (5) is the solution to the differential equation (4). Hint: this is similar to a question in Lab M1, The Simple Pendulum.

2. A parallel-plate capacitor has area $A = 500 \text{ cm}^2$, separation $d = 0.550$ mm, and is filled with a dielectric medium with $\kappa = 8.40$. What is the capacitance of the this capacitor, in pF?

3. In part II of this lab, you will graph ln(V) vs. t for an RC circuit. What should this graph look like? No numbers! - just a qualitative sketch. In terms of R and C, what is the slope of this graph?

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4. Does the capacitance C of a capacitor depend on its charge Q? Explain.

5. In part I of this lab, you are to plot C vs. 1/d for a parallel plate capacitor. What should this graph look like? No numbers — just a qualitative sketch! If this graph was a plot of data from a capacitor with a pure air gap (no dielectric of any kind anywhere), what would be the slope of this graph?

6. A capacitor C_1 has 3 times the capacitance of a second capacitor C_2 : $C_1 = 3\,C_2$. The same voltage V is applied to both capacitors. What is the ratio of the charges Q_1 / Q_2 on the two capacitors?

7. The program linfit.mcd yields the slope $m \pm \delta m$ of the plot ln(V) vs. t. Show how to compute τ and $\delta\tau$, given m and δm.

8. In part I of this lab, what is the stray capacitance and how is it measured?

9. An RC circuit has a time constant of $\tau = 85\,\text{msec}$. A switch is closed, which causes the capacitor to discharge through the resistor. How long after the switch is closed will the voltage on the capacitor have fallen to 2% of its initial value?

10. True or False: if one capacitor is physically larger than another capacitor, then the larger-sized capacitor must have a larger capacitance.