

### Lab E2: B-field of a Solenoid

In this lab, we will explore the magnetic field created by a solenoid. First, we must review some basic electromagnetic theory. The magnetic flux over some area  $A$  is defined as

$$(1) \quad \Phi = \int_A \vec{B} \cdot d\vec{a}$$

In the case that the B-field is uniform and perpendicular to the area, (1) reduces to

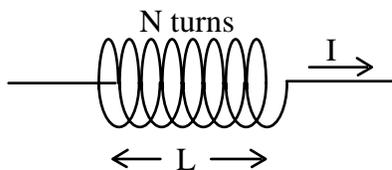
$$(2) \quad \Phi = B \cdot A.$$

Faraday's Law relates the time rate of change of the flux,  $\frac{d\Phi}{dt}$ , to the emf  $\mathcal{E}$ :

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \left( \int \vec{B} \cdot d\vec{a} \right)$$

(3)

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

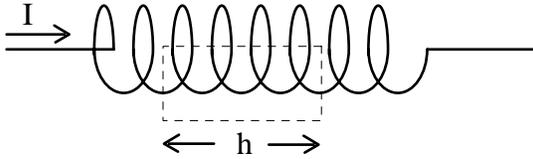


Electrical currents create magnetic fields and the simplest way to create a uniform magnetic field is to run a current through a solenoid of wire. Consider a solenoid of  $N$  turns and length  $L$ , carrying a current  $I$ ; the number of turns/length is  $n = N/L$ .

For a long solenoid of many turns, the B-field within is nearly uniform, while the field outside is close to zero. The field within the coil can be calculated from Ampere's Law:

$$(4) \quad \oint \vec{B} \cdot d\vec{s} = \mu_0 I,$$

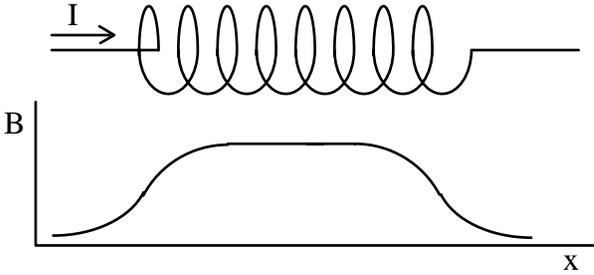
which, in words, is: the line integral of  $\vec{B}$  around a closed loop is proportional to the current through the loop. The constant  $\mu_0$  is called the permeability constant and has the value  $4\pi \times 10^{-7}$ , in SI units. To compute the field inside the coil, we draw an imaginary square loop with one side inside the solenoid and one side outside, like so:



Applying Ampere's Law to this loop, we have

$$B \cdot h = \mu_0 n h I, \text{ or}$$

$$(5) \quad B = \mu_0 n I.$$



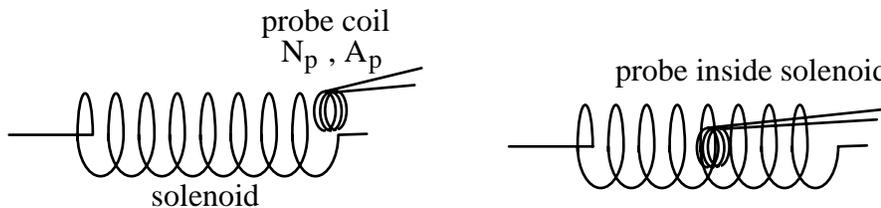
Eq'n (5) is really only valid for an infinitely long solenoid with closely spaced turns. Near the ends of a finite-length solenoid, the field is smaller than predicted by (5); at the ends of a finite solenoid,  $B$  is  $1/2$  its maximum value. The factor of  $1/2$  can be understood from a symmetry argument: the end of a

solenoid can be thought of as an infinite solenoid with one half cut away. In an infinite solenoid, the total  $B$ -field at any point is due to the current in the coils to the left and the current in the coils to the right. By removing the coils on one side, the  $B$ -field is cut in half.

If an AC current flows through the solenoid, then it will generate an AC<sup>1</sup> magnetic field, which can be easily probed with a second coil of wire, called a probe coil or pick-up coil. If the probe coil has  $N_p$  turns and an area  $A_p$ , then by Faraday's Law (3), the emf induced in the probe coil due to the  $B$ -field from the solenoid is

$$(6) \quad \varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}(N_p \cdot B \cdot A_p) = -(N_p A_p) \cdot \frac{dB}{dt}.$$

In writing (6), we have assumed that the probe coil is stationary and perpendicular to the  $B$ -field.



If the AC current in the solenoid has a frequency  $f$ ,

$$(7) \quad I(t) = I_0 \sin(2\pi f t) = I_0 \sin(\omega t),$$

<sup>1</sup> The term AC is short for "alternating current"; however, in physics and engineering, the term has come to mean "time-varying", usually sinusoidal.

then, from (5), the B-field at the center of the solenoid is

$$(8) \quad B(t) = \mu_0 n_s I_0 \sin(\omega t) = B_0 \sin(\omega t),$$

where  $n_s$  is the number of turns/length in the solenoid, and  $B_0$  is the amplitude of the field. The resulting emf in the probe coil is

$$(9) \quad \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}[N_p \cdot A_p \cdot B_0 \sin(\omega t)] = -\omega \cdot N_p \cdot A_p \cdot B_0 \cos(\omega t) .$$

Thus the probe coil will produce an AC emf whose amplitude  $\mathcal{E}_0$  is proportional to the frequency  $\omega$  and the amplitude  $B_0$  of the AC B-field:

$$(10) \quad \mathcal{E}_0 = \omega \cdot N_p \cdot A_p \cdot B_0 .$$

If the probe coil is deep inside a solenoid and the field  $B_0$  is given by (5), then we have:

$$(11) \quad \mathcal{E}_0 = 2\pi f \cdot N_p \cdot A_p \cdot \mu_0 \cdot n_s \cdot I_0 ,$$

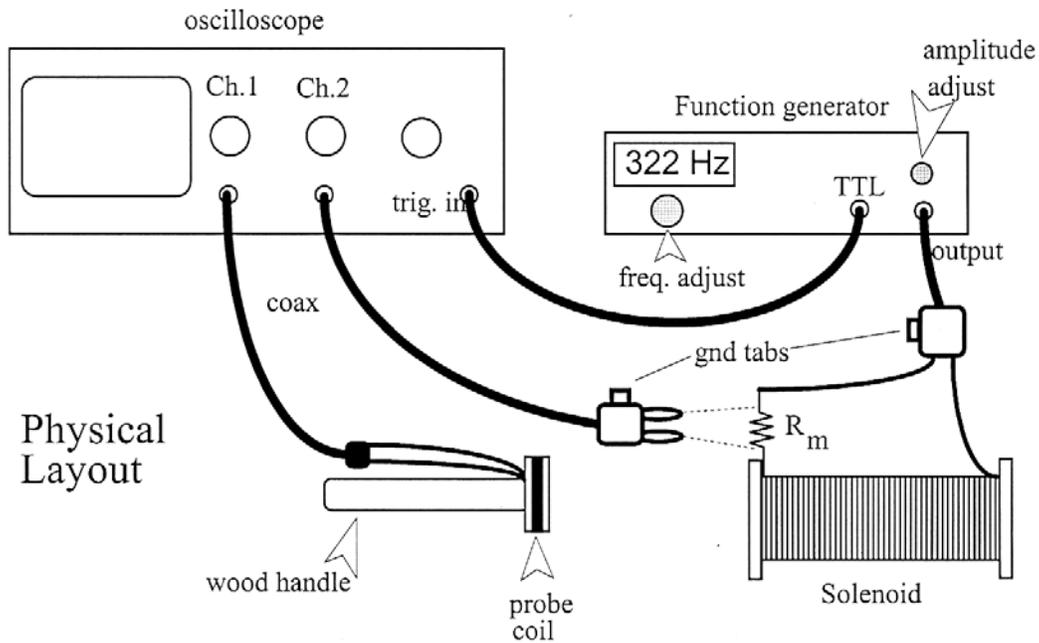
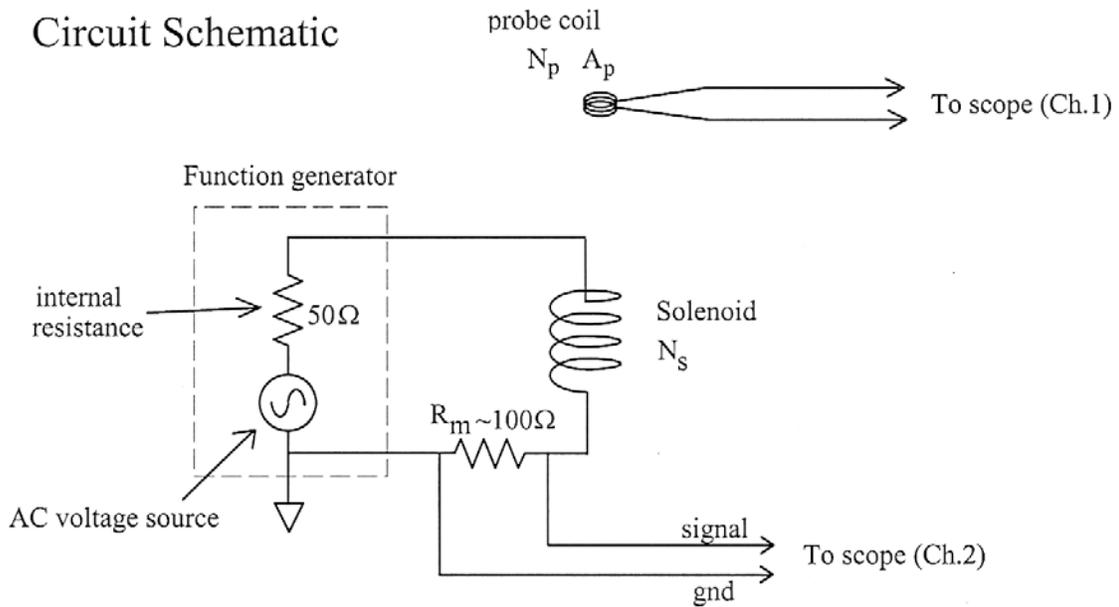
where  $2\pi f = \omega$ ,  $n_s$  is the number of turns/length of the solenoid, and  $I_0$  is the amplitude of the AC current. We see that the emf of the probe coil is proportional to both the frequency  $f$  and the amplitude  $I_0$  of the AC current in the solenoid.

## Experiment

A circuit diagram of the experiment is shown below. The function generator produces a sine-wave AC voltage which drives an AC current through the solenoid. The emf produced in the probe coil by the B-field from the solenoid is measured with the oscilloscope. The AC current through solenoid is obtained by monitoring the AC voltage  $V_m$  across a monitor resistor  $R_m$ . (It's called a monitor resistor because its only function is to monitor the current.) The current through the solenoid is  $I = \frac{V_m}{R_m}$ .

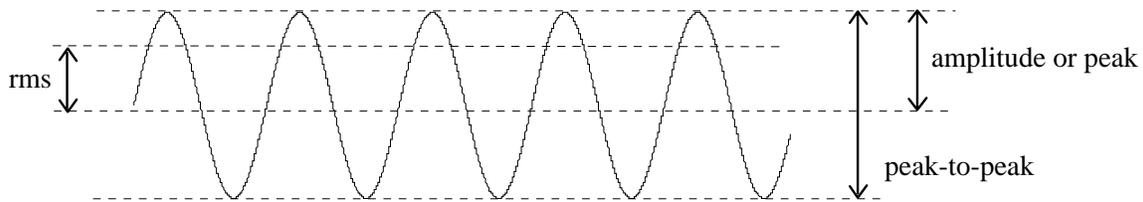
The solenoid has  $N_s =$  a few hundred turns and an inductance of a few mH [a henry (H) is the SI unit of inductance]. The exact values of  $N_s$  and other parameters are given at the experimental station. The probe coil has  $N_p =$  a few thousand turns, an effective area of  $A_p =$  a few  $\text{cm}^2$ , and an inductance of roughly 0.5 H. The function generator has a  $50\Omega$  internal resistance (Such an internal resistance is called the output impedance. Although this is shown in the circuit schematic, it will not affect your measurements.)

## Circuit Schematic



When checking the connections, it is important to remember that the outer conductor of the coaxial cables to the oscilloscopes must be at 0 volts (ground) and the ground side of the double-banana connectors is indicated by a little plastic tab.

In this lab, we will be measuring AC voltages with the oscilloscope. There are at least three ways to indicate the magnitude of an AC signal: peak-to-peak, peak or amplitude, and rms amplitude.



The rms or root-mean-square amplitude is given by  $\sqrt{\langle V^2 \rangle}$ , where the brackets  $\langle \rangle$  indicate an average. The voltage  $V$  alternates (+) and (-) so the average voltage  $\langle V \rangle$  is zero, but the voltage-squared  $V^2$  is always positive so the average of  $V^2$  is non-zero and positive. If  $V(t)$  is a sinusoid, then it turns out that the rms amplitude  $V_{\text{rms}}$  and the amplitude  $V_o$  are related by  $V_o = \sqrt{2} V_{\text{rms}}$ . Almost all instruments which measure AC voltage or current, including our DMM's, display rms values. In this lab, you will be reading the voltage directly from the oscilloscope screen, so you should indicate whether you are reporting the amplitude or peak-to-peak voltage. It would be nice if we could use our DMM's to measure the AC voltages directly without having to read them from the oscilloscope screen. Unfortunately, our DMM's only read AC signals accurately if the frequency is less than 1kHz, and we will be making measurements at higher frequencies.

## Procedure

Before beginning the experiment, you should familiarize yourself with the equipment. Plug the output of the function generator directly into the oscilloscope and play with all the knobs until you are comfortable with the controls. Don't be afraid to touch knobs; you can't break anything here and you will only learn by doing. Remember to check that the CAL knobs on the oscilloscope are fully CW. (Look at Lab E1 if you don't remember what the CAL knobs do.)

Begin by measuring the length  $L$  of the solenoid with the plastic calipers or with a meter stick and compute the turns per length  $n_s$  of the solenoid. Then use the digital multimeter (DMM) to measure the resistance  $R_m$  of the current-monitor resistor. It is important to temporarily disconnect the resistor from the rest of the circuit while you are measuring its resistance with the DMM.

## Magnitude of the B-field: Theory and Experiment

After constructing the circuit, set the frequency to around 1 kHz and adjust the amplitude of the function generator output until the amplitude of the monitor voltage  $V_m$  is about 2.0 V. This produces a current with an amplitude of

$$I_o = \frac{V_m}{R_m} \cong \frac{2V}{100\Omega} = 20\text{mA} . \quad \text{Place the probe coil in the center of the solenoid and}$$

observe the AC voltage on the probe coil produced by the B-field of the solenoid. Move the probe coil all around the solenoid, inside and out, and observe the changes in the signal.

Measure the voltage (the emf) on the probe coil when it is in the center of the solenoid. Using eq'n (10), compute the B-field measured by the coil. Call this  $B_{\text{meas}}$ . Now using eq'n(5), compute the B-field produced by the solenoid. Call this  $B_{\text{calc}}$ . Compare the two. Do they agree within uncertainties?

## Position Dependence of the B-field

Measure the ratio of the B-field at the edge of the solenoid to the B-field at the center  $\left( \frac{B_{\text{edge}}}{B_{\text{center}}} \right)$ . Keep in mind that the B-field from the solenoid is proportional to the emf in the probe coil [eq'n (10)], so you should *not* have to compute  $B_{\text{edge}}$  and  $B_{\text{center}}$  separately to get the ratio. Compare your answer with theory.

As one moves along the axis of the solenoid away from its center, the B-field drops continuously and is very small at positions outside the solenoid and far from its edge. Make measurements to determine the position along the axis at which the B-field is 1% of its maximum value at the center. (Computing this position is quite messy; just measure it.) Express this distance (from the edge of the solenoid to the center of the probe coil) in cm and in solenoid diameters.

## Frequency Dependence of the emf

Measure the frequency dependence of the emf in the probe coil by varying the frequency while keeping the AC current in the solenoid constant<sup>2</sup>. For several frequencies from about 30Hz up to 10kHz, measure the amplitude of the coil emf on Ch.1 of the oscilloscope. When measuring a variable over several decades (several powers of 10), a good strategy for picking points is to approximately double the variable each time:

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<sup>2</sup> The "impedance"  $Z$  of the solenoid (impedance = effective resistance to the flow of current) is frequency-dependent:  $Z = 2\pi f L$ , where  $L$  is the inductance of the solenoid. Thus, as the frequency is varied, the current through the solenoid will vary if the AC voltage from the function generator is kept constant. So, to maintain constant current, the voltage must be adjusted as the frequency is varied.

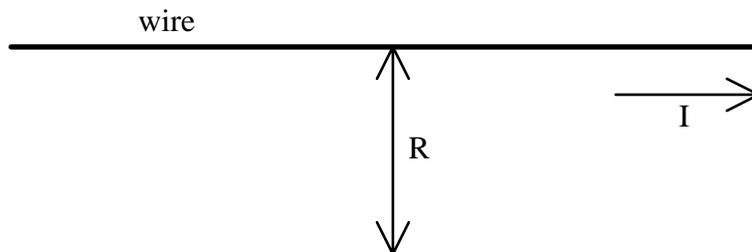
30Hz, 60, 100, 200, 500, 1000, ... At each frequency, before measuring the emf, set the current to a constant value by setting  $V_m$ , read on Ch.2 of the scope, to 2.00 V amplitude. Remember to always adjust the volt/div knob and the position of the trace on the screen so that the signal fills as much of the screen as possible and you make measurements with maximum precision. [Do not take measurements above 10kHz. At frequencies  $f > 10\text{kHz}$ , the inter-coil capacitance of the probe coil becomes significant and it no longer acts like a pure inductance. It resonates like an L-C circuit at about 20kHz.]

Plot the measured emf vs. frequency  $f$ . Make both a linear plot and a log-log plot. Use the X-Y Plot\Format menu to set both x- and y-axes scales to logarithmic. On the same graphs, plot the curve predicted by theory, Eqn.(11). Do the data agree with theory?

Finally, pretend that the product  $N_p \cdot A_p$  is unknown to you, and use eq'n(11) and your measurements to compute the product  $N_p \cdot A_p$  for each frequency. Compute the average value and the uncertainty of  $N_p \cdot A_p$ . [Recall that the uncertainty of an average is the standard deviation of the mean.] Compare your result with the "known" value of  $N_p A_p$ .

### Prelab Questions

1. A long narrow solenoid with  $N = 500$  turns and length  $L = 20.0\text{cm}$  has an AC current with amplitude  $I_0 = 0.100\text{A}$  and frequency  $f = 2000\text{ Hz}$ . What is the *amplitude* of the B-field at the center of the solenoid? What is  $B = B(t)$ , the time-dependent B-field? [Careful about units: it is best to work entirely in the SI (also called the MKS = meter-kilogram-second) system when working electric circuit problems.]
2. A probe coil with  $N = 8000$  turns and  $A = 1.00\text{ cm}^2$  is placed in the center of the solenoid of question 1. The probe coil is oriented so its emf is maximum. What is the *amplitude* of the emf? What is the frequency of the emf?
3. Show that you understand how to apply Ampere's Law by using it to compute the magnitude of the B-field a distance  $R$  from a long straight wire carrying a current  $I$ .



4. In this experiment, how is the current through the solenoid measured?
5. The AC voltage of a wall socket has a frequency of 60Hz and an rms amplitude of 120V. What is the peak value or amplitude of this voltage?
6. The earth's magnetic field is about 0.5 gauss. The SI unit of magnetic field is the tesla (T) [1 Tesla = 10,000 gauss] . A probe coil with  $N = 10,000$  turns and area  $A = 10 \text{ cm}^2$  is oriented with the coil perpendicular to the earth's field. Is there an emf induced in the coil? If so, how big is it? If not, why not?
7. A probe coil with  $N_p$  turns and area  $A_p$  is placed in the center of a solenoid with  $n_s$  turns per unit length and carrying an AC current of amplitude  $I_o$  and frequency  $f$ . What happens to the emf in the probe coil if all five of the quantities,  $N_p$  ,  $A_p$  ,  $n_s$  ,  $I_o$  , and  $f$  are tripled?
8. Consider the function  $f(x) = m \cdot x$  , where  $m$  is a positive constant. How is  $\log f$  related to  $\log x$ ? [That is, write an equation containing  $\log f$  and  $\log x$ , showing how they are related.] Make a sketch of the graph  $f$  vs.  $x$  and a sketch of the graph  $\log f$  vs.  $\log x$ . Indicate clearly the slope and intercept of the two graphs.
9. True or False: A constant magnetic field in a stationary probe coil will produce an emf.
10. True or False: It is perfectly safe to place your tongue in a 120V electrical outlet.